Word Problems 8: The Mixed-Rate Problems #5

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Abstract

In this algebra word problem note, we use the Scheme to solve our fifth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Word Problem #8.1

In what ratio should water be added to a liquid costing \$12 per liter so as to make a profit of 25% by selling the diluted liquid at \$13.75 per liter?

2 Solution 8.1.1: Conceptualizing the problem

We'll worry about the ratio after we have calculated how much water should be added to the starting liquid, which we'll set at 1 liter. We lose no generality by doing this.

But first, a word about this 25% profit. How do we deal with it? Percentage changes to a decimal 0.25 as a multiplier. I won't go into detail because I offer this only as a refresher to what the reader is presumed already familiar with.

$$\begin{aligned}
\text{(retail cost)} &= \text{(base cost)} + \text{(profit)} \\
&= \text{(base cost)} + (0.25)\text{(base cost)} \\
&= (1.25)\text{(base cost)}
\end{aligned}$$

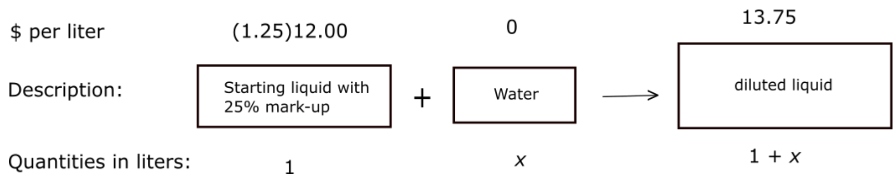

Figure 1. This graphic represents the adding some water x to a starting liquid in a 'before and after' process. The arbitrary markup has already been applied to the cost per liter before adding water.

In the graphic in Figure 1, we show a 'before and after' process of adding water to this starting liquid. We begin with the conservation equation

We begin with the cost conservation equation:

$$(1.25)(\$12.00/\text{liter})(1\,\text{liter}) + (\$0.00/\text{liter})(x\,\text{liter}) = (\$13.75/\text{liter})(1+x)\text{liter}\,. \tag{1}$$

Solving this, $x = 0.090909....$ But we are asked to find the ratio of $x:1$, which is $0.090909:1$, or (approximately) $1:11$.

3 Word Problem #8.2

A merchant has 100 lbs of sugar, part of which (x lbs) he sells at 7% profit and the rest (y lbs) at 17% profit. The division of the whole into two parts is to be made so that the net profit is the same as 10% on each original quantity of sugar. How much is each part?

4 Solution 8.2.1: Conceptualizing the problem

Let's begin with a figure to help us conceptualize the data.

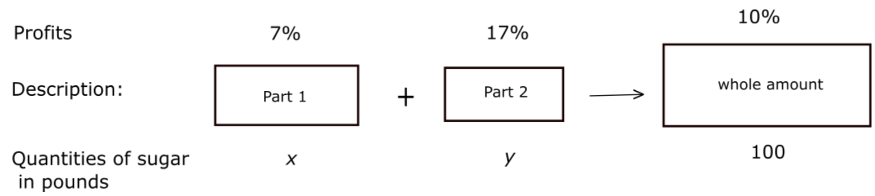

Figure 2. How to divide 100 pounds of sugar to get 10% profit.

So, we have two equations in two unknowns, beginning with the conservation of weight of sugar (in pounds):

$$x + y = 100\,. \tag{2}$$

And we have the conservation of profit:

$$(\text{profit off of } x) + (\text{profit off of } y) = (\text{net profit off of 100 lbs})\,. \tag{3}$$

For the next refinement, we'll convert percentages to decimals and multiply rates times quantities off Figure 2, to get

$$.07x + .17y = .10 \cdot 100 = 10.00\,. \tag{4}$$

Solving (2) and (4) together yields $x = 70$ and $y = 30$ in pounds.

5 Word Problem #8.3

Two vessels A and B containing milk and water in ratios 4 : 3 and 2 : 3, respectively. In what ratio should they be added together so that their final mixture is in ratio 1 : 1?

6 Solution 8.3.1: Conceptualizing the problem

Notice in Figure 3 that we used arbitrary volume units. One reason for this is that we weren't given a specific unit to work with, and the other is that we can choose any particular unit we please because in taking ratios the units will cancel.

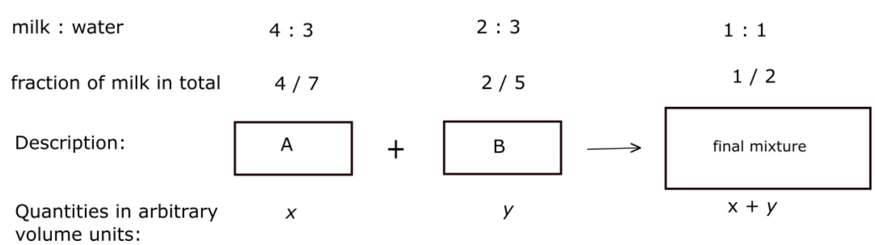

Figure 3. We need to solve for the ratio of x and y.

Now, we've already shown the conservation of volume in the bottom line. We need now only one more equation in x, y to solve for their ratios (hopefully). For that, we show the conservation of milk on both sides.[1]

$$(\text{milk in } A) + (\text{milk in } B) = (\text{milk in final mixture})\,. \tag{5}$$

7 Solution 8.3.2: Solving the problem

From (5) we get

$$\frac{4}{7}x + \frac{2}{5}y = \frac{1}{2}(x+y)\,. \tag{6}$$

The variable we need to solve for is x/y, and to do this efficiently, let's divide the last equation through by y, to get

$$\frac{4}{7}x/y + \frac{2}{5} = \frac{1}{2}(x/y+1)\,. \tag{7}$$

Let's make one more simplification and substitute $\lambda = x/y$ to get

$$\frac{4}{7}\lambda + \frac{2}{5} = \frac{1}{2}(\lambda+1)\,, \tag{8}$$

with solution $\lambda = 7/5$. Therefore $x : y :: 7 : 5$.

[1] The figure was setup to show the conservation of milk, but we could just as easily have shown the conservation of water.