Word Problems 29

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Abstract

In this algebra word problem note, we use the Scheme to solve a variety of problems. This paper introduces the novel **Extract function**, that receives an equation and solves it for whatever it needs to solve for. See the third problem for the novelty of how this function is used as a heuristic.

1 Introduction

Sometimes it's better to use a box diagram. The first problem is a case in point, where we use a foursquare.

2 Word Problem #29.1

[1] At a technology consulting firm with x computers, all of which are laptops or desktops, 30% are laptops; if 80% of the total number of computers have more than 1GB of RAM and 10% of the computers with less than 1 GB of RAM are laptops (and no computers have exactly 1GB of RAM), approximately what percent of desktops have more than 1GB of RAM?
A) 75%
B) 60%
C) 52%
D) 40%
E) 45%

3 Solution 29.1.1: Conceptualizing the Problem

Whoo! This problem has enough information for two diagrams, where you add parts to get their totals. Exactly. To accommodate this information, we'll employ two figures collapsed into one in the form of a foursquare. See Figure 1.

[1]Found at http://www.beatthegmat.com/650-800-question-t68610.html.

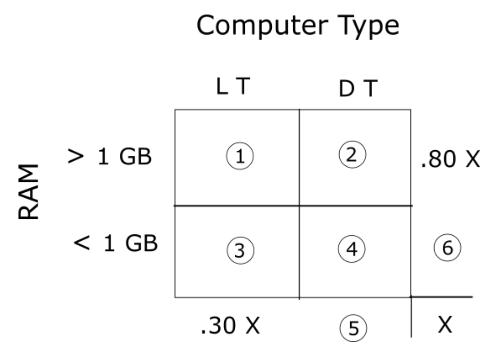

Figure 1. We start the process of constructing the foursquare by first noting that the bottom row and the rightmost column must each separately add to X.

Circle 5 and Circle 6 can be determined, respectively, by making the bottom row and right column add to X.

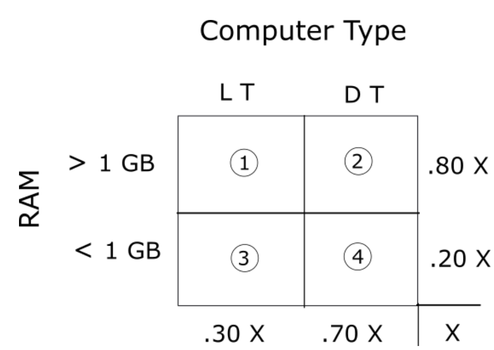

Figure 2. We are told that 10% of the computers with less than 1 GB of RAM are laptops. With this information we can calculate what should go into Circle 3.

The value of Circle 3 is the probability of being less than 1 GB and being a laptop. We can use conditional probabilities to calculate this value.

$$\begin{aligned}\Pr((\text{L T}) \cap (< 1 \text{ GB RAM})) &= \Pr((\text{L T}) \,|\, (< 1 \text{ GB RAM}))\, \Pr(< 1 \text{ GB RAM}) \\ &= (.10)(.20) = .02\,. \end{aligned} \tag{1}$$

Computer Type

		L T	D T	
RAM	> 1 GB	(1)	(2)	.80 X
	< 1 GB	.02 X	(4)	.20 X
		.30 X	.70 X	X

Figure 3. Really — a separate figure just for one square? Yes. Figuring out what should go into Circle 3 is the crux of finishing off the foursquare.

We can now fill in the rest of the foursquare on the principle the values in the rows of the foursquare must add to the subtotals on the right, and that the values in the columns of the foursquare must add to the subtotals in the bottom of the foursquare. We can start with finding either the values that belong in Circle 1 or Circle 4.

Computer Type

		L T	D T	
RAM	> 1 GB	.28 X	.52 X	.80 X
	< 1 GB	.02 X	.18 X	.20 X
		.30 X	.70 X	X

Figure 4. With all the values inside the foursquare computed, we can now now answer the question asked of us.

So, we were asked this question: Approximately what percent of desktops have more than 1GB of RAM? To which we can report:

$$\Pr((\text{D T}) \cap (> 1 \text{ GB RAM})) = .52\,, \tag{2}$$

which, on converting to a percentage, becomes 52%.

4 Word Problem #29.2

[2] If jogging for 1 mile uses 150 calories and fast walking for 1 mile uses 100 calories, a jogger has to go how many times as far as a walker to use the same number of calories?

5 Solution 29.2.1: Conceptualizing the Problem

One equation jumps out at us: The number of calories used by the jogger = the number of calories used by the walker. Let D_J be the distance the jogger must run to use the same number of calories the walker will use in distance D_W.

Setting up the equation, we get

$$D_J \left[\frac{150\ \text{cal}}{\text{mile}}\right] = D_W \left[\frac{100\ \text{cal}}{\text{mile}}\right] . \tag{3}$$

On simplifying, we get

$$D_J = \frac{2}{3}\, D_W \, . \tag{4}$$

Thus, the jogger needs to go only two-thirds the distance the walker needs to go to burn-up the same number of calories.

6 Word Problem #29.3

Question 123779:[3] A rock is dropped from a cliff into the ocean. It travels $16t^2$ feet in t seconds. If the splash is heard 1.5 second later, how high is the cliff? [Note: Assume the speed of sound at sea level is 1100 feet per second.]

7 Solution 29.3.1: Conceptualizing the Problem

Let h be the height of the cliff and v be the speed of sound. The total time Δt from when the rock is released until the splash is heard is given as the sum (suppress units)

$$\begin{aligned}
\Delta t &= \begin{bmatrix} \text{time for rock to} \\ \text{hit the water} \end{bmatrix} + \begin{bmatrix} \text{time for sound of splash} \\ \text{to hit the ears} \end{bmatrix} \\
&= \text{Extract_Time}[h = 16t^2] + \text{Extract_Time}[t = h/v] \\
&= \frac{\sqrt{h}}{4} + \frac{h}{1100} \, .
\end{aligned} \tag{5}$$

But $\Delta t = 1.5$ seconds. So, letting $z \equiv \sqrt{h}$ and clearing of fractions, we get

$$z^2 + 275z - 1650 = 0 \, , \tag{6}$$

[2]From *Nursing School Entrance Exam*, 2005, LearningExpress, p. 52.
[3]Found at https://www.algebra.com/algebra.

with solution $z \approx 5.8745$. That gives us $h \approx 34.5$ feet.

Thus we see that the Extract_X function inputs an equation and returns the expression equivalent to X.