Word Problems 19: Mixed-Rate Problems #16

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Abstract

In this algebra word problem note, we use the Scheme to solve our sixteenth attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

Some more tricky problems, this time dealing with ambiguous use of percentages when framing the problem. Wikipedia tells us that the notion of a "percent solution" is (often) ambiguous. When looking on the Internet for so-called 'mixture problems' to solve (I refer to them as 'mixed-rate problems'), I often come across a problem posed like this: *A chemist wants to make a 15% solution of X using a 5% solution and a 20% solution.* And it is often ambiguous what is meant.

This is especially true when the units are given in volumes rather than in weight or mass. One case in which it is all right to use volumes is units in percentage problems, is when dealing with dry goods that can be effectively measured by volume, or by when dealing with pure fluids that are miscible together.

Before we do the first problem on percentages, we must ask: What *really* is a percentage? The answer is simple: A *percentage* is the ratio of a part to its whole times 100%, and therefore is a unitless number.

2 Word Problem #19.1

This first word problem is from the PDF handout found at

`http://www.weber.edu/wsuimages/MTC/Handouts/Mixture %20Problems%20Handout_WEllis.pdf`

The first problem posed is useful for two reasons: First, because it uses a graphic similar to what I recommend in the Scheme, and second, because the posed problem is an unambiguous use of percentages.

Problem Statement:

> Suppose a chemist has 15 L of a 40% HNO_3 solution. He needs to know how much of an 85% HNO_3 solution he would have to add to the 40% solution to get a 57% HNO_3 solution. Round the answer to the nearest hundredth if necessary. Let's look at a method to organize this information. One way is to draw out the beakers the chemist is using, like this:

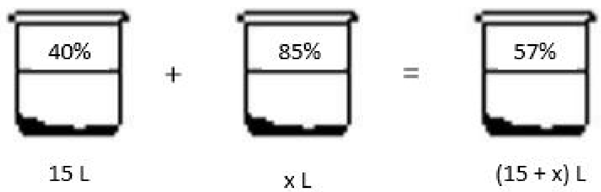

Figure 1. Figure accompanying the problem found in the handout on mixture problems.

3 Solution 19.1.1: Conceptualizing the Problem

I'm not actually going to solve this problem, as we have already seen many similar problems solved to date. However, I do want to state that the figure is correct and useful, though the format is a bit different (and less informative) from that proposed in Scheme. However, the equation the author used to solve the problem is derived from an unambiguous use of the information. Pure HNO_3 is a liquid so its percentage in the fluid is a ratio of its volume to the volume of the fluid, assuming that is how the acid mixture were actually made. With that assumption, we can write the 'conservation of acid' equation

$$.40(15) + .85(x) = .57(15 + x)\,. \tag{1}$$

But if those percentages given on the beakers were actually made by mixing-up the solution percentages by weight, then this calculation is wrong.

So, what makes me suspicious of this methodology with percentages of contents for fluids? For one, even when two liquids are mixed, they could be mixed by weight, such as when 3% hydrogen peroxide is made (according to Wikipedia). Naively assuming you can make these simplifying assumptions may not harm anyone on a math test, but if used improperly in the real world, it could be disastrous.

In the real world of chemistry, for example, the standard way to measure concentration is by molarity, or moles per liter, and *not* usually by percentages.

4 Word Problem #19.2

On the next page of the PDF handout, we find this problem:

> Suppose a store keeper wants to make a mixture of cashews and peanuts. He has on hand peanuts that cost $3 per pound and cashews that cost $5.50 per pound. He wants to make a 3 pound mixture that costs $4 per pound.

And accompanying figure

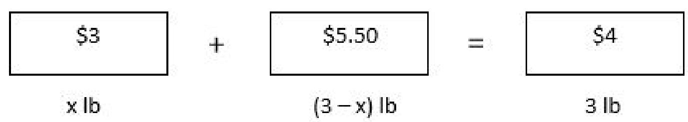

Figure 2. Figure was imported from word problem in handout.

5 Solution 19.2.1: Conceptualizing the Problem

By contrast, the equivalent figure suggested by Scheme would be:

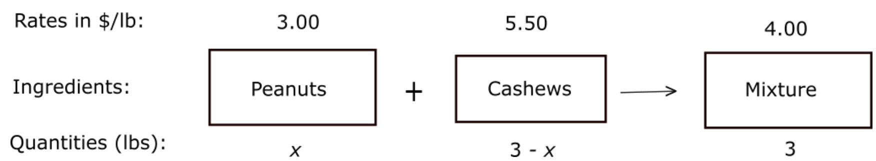

Figure 3. The Scheme version of the previous figure.

In a standard Scheme figure, rates are on top, quantities are on the bottom, and names, or identifiers, are inside the rectangles.

6 Word Problem #19.3

Question 702388.[1] if 20 pounds of sea water contains 1.6 pounds of salt, how many pounds of pure water must be added to produce a mixture containing 5% salt?

7 Solution 19.3.1: Conceptualizing the Problem

Another easy problem once we identify the totals and their parts. The point of me showing it, though, is to show a percent problem that makes sense and can be solved. Why? Because all the quantities are given in terms of pounds, so percent becomes a unitless quantity. (See the next page for the figure.)

[1]Found at https://www.algebra.com/algebra.

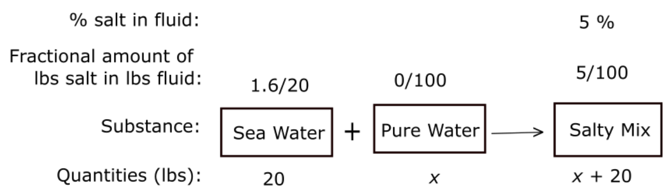

Figure 4. We've already balanced on pounds overall.

When we balance on salt over the before-and-after process, we'll get the equation we need to solve for x:

$$\frac{1.6}{20}\,20 + 0x = \frac{5}{100}\,(x + 20)\,. \tag{2}$$

This equation has solution $x = 12$ pounds.

8 Word Problem #19.4

Question 15639:[2] This problem has me stumped because I have 8 ounces already mixed and need to add to it to get 12 ounces. If I was just adding enough chocolate syrup to make it 42 percent, I would not be so confused. Here is the problem: I want to make the perfect 12 ounce cup of chocolate milk. It requires that the mixture is 42% chocolate syrup. What I have right now is 8 ounces of milk/syrup mixture that I know contains 30% syrup. What must the syrup concentration be in the remaining mixture that I must add in order to achieve perfection?

9 Solution 19.4.1: Conceptualizing the Problem

The reason this problem solver was so confused was because he or she did not understand the basics of solving mixed-rate problems. Adding chocolate syrup to chocolate milk is a 'before and after process' that preserves (conserves) the total amount of both syrup and milk in the doing

So, to choose one to balance on, we can write down the simple equation that the total amount of chocolate in the constituent parts (the original chocolate milk and the chocolate syrup) equals the total mount of chocolate in the end product, the super-chocolate milk. We need just one equation, which we'll get from balancing on chocolate (the ounces, that is) on both sides:

$$(.30)8 + (P/100)4 = (.42)12\,, \tag{3}$$

where we converted percentages into decimal values or decimal equivalent values. Equation (3) has solution $P = 66$.

[2] Found at https://www.algebra.com/algebra.

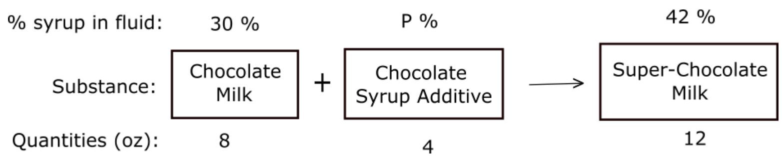

Figure 5. Standard setup for calculation: We've already shown the conservation of overall volumes in ounces.

10 Word Problem #19.5

Question 22541:[3] Help! I can't even think of a good equation for this problem! Brine is a solution of salt and water. If a tub contains 50 pounds of a 5% solution of brine, how much water must evaporate to change it to an 8% solution? Any help would be greatly appreciated!

I can tell you right off what a really good equation for this problem is: Every total is equal to the sum of its parts! Here's a good equation formed from that kind of analysis:

$$\text{(Wt. of brine before evap.)} = \text{(Wt. of brine after evap.)} + \text{(Wt. of water evap.)} \quad (4)$$

In the early stages of analysis of a mixture problem, a bad question to ask yourself is this: *I know what the unknown is. How do I relate the unknown to the given information?* Experience strongly suggests that the right questions to ask are these:

- What are the totals?
- What are the parts?
- What are the conserved quantities of a before-and-after process?

11 Solution 19.5.1: Conceptualizing the Problem

At least this problem also gives its data in terms of weights, so percentages make sense here, too.

We start with a tub with 50 pounds of brine in it. We will let some unknown amount x evaporate from it, leaving $50 - x$ pounds of concentrated brine in the tub. This is an equation! Let's show the process in a figure.

[3]Found at https://www.algebra.com/algebra.

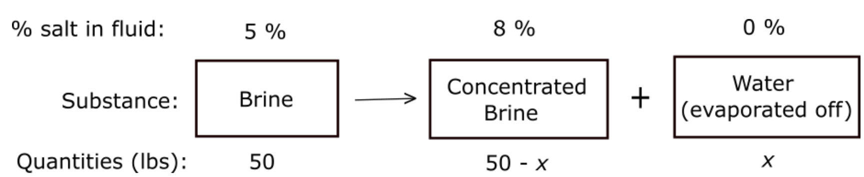

Figure 6. Standard setup for calculation: We've already shown the conservation of overall fluids in pounds.

We need just one equation, which we'll get from balancing on salt on both sides:

$$(.05)(50) = (.08)(50 - x) + 0x\,. \tag{5}$$

which has solution $x = 18.75$ pounds.

12 Word Problem #19.6

Question 494329[4]: How many liters of an 8% solution of salt should be added to a 25% solution in order to obtain 510 liters of an 12% solution?

I'm not going to solve this problem because it makes no sense to me. However, problems like this are given routinely for students to solve as though they do make sense.

The problem is that a percentage is unitless. Therefore, when taking the ratio of the 'part to the whole' (salt part/whole fluid)$\times$100%, in order to make the fraction unitless, since the whole is given to us in liters, we are forced to interpret the salt as also being in liters. Really? Does this makes sense? I say that for general purposes, it does not. On top of that, there is a limit to how much salt can be dissolved in water.

What if we designate x as the amount in liters of 8% salt solution we need to add to $(520 - x)$ liters of 25% solution to get 510(.25) liters of solution. As an equation we write

$$(.08)(x) + (.25)(510 - x) = (.12)(510)\,, \tag{6}$$

where each term has unit of liters. Crazy! We've just claimed that salt comes in liters.

Now, we *can* do percentages with volumes, but we have to be careful. For example, we can talk about percentages by volumes of alcohol in an alcohol-water mixture. And we can talk about percentages by volumes of oil in an oil-gasoline mixture, say. In fact, mixing oil and gasoline by volumes is probably the better way to do it for the homeowner making a proper mix for his home gasoline-powered lawn mower. By the way, a US *fluid ounce* is not a weight. It is a volume, being 1/128 th part of a gallon.

[4]Found at https://www.algebra.com/algebra.

13 Word Problem #19.7

Question 521012: Two Kleaning ladies company needs a 50% bleach mixture solution. they make 12 liter at a time from a 40% bleach solution and a 70% bleach solution. How many liters do they need of each?

14 Solution 19.7.1: Conceptualizing the Prob.

A common sense way to make a 50% bleach mixture is by adding one part of bleach to one part of water. A 40% bleach mixture can be made directly by adding 4 parts of bleach to 6 parts of water. And, 70% bleach mixture can be made directly by adding 7 parts of bleach to 3 parts of water.

But our clever cleaners want to redeem some of the bleach mixes they already have on hand.

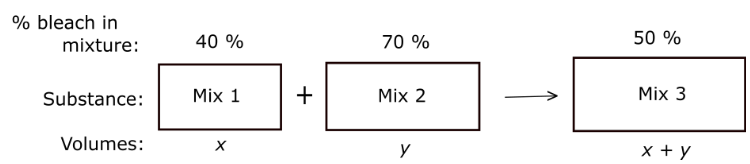

Figure 7. We've already shown the conservation of volumes.

Balancing for bleach, we have that

$$(.40)(x) + (.70)(y) = (.50)(x + y)\,, \tag{7a}$$

which can be simplified by the substitution $\lambda = x/y$ and multiplying through by 10, yielding us

$$4\lambda + 7 = 5\lambda + 5\,, \tag{7b}$$

which has solution $\lambda = 2$. Therefore, for every 2 parts of the 40% solution we add 1 part of the 70% solution to obtain 3 parts 50% solution. Now, let's prove that the ratio of bleach to water in Mix 3 is 1:1.

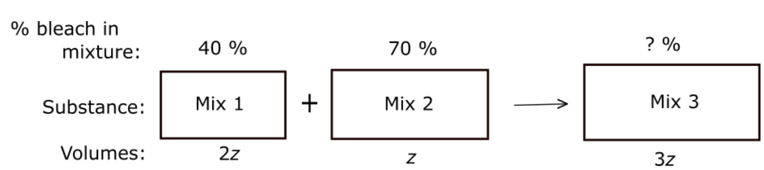

Figure 8. We have redone the last figure with the required ratio $x : y$ included for us. z is an arbitrary positive number of any volume unit.

Balancing for bleach, we have that

$$(.40)(2z) + (.70)(z) = 1.5z\,, \tag{8a}$$

and balancing for water, we have that

$$(.60)(2z) + (.30)(z) = 1.5z\,. \tag{8b}$$

Therefore, on taking the ratio of bleach to water, we get 1 : 1, which is what we claimed was the correct way to make 50% bleach from scratch.

15 Word Problem #19.8

Question 508314: The dosage of a medicine ordered by a doctor is 40 mL of a 16% solution. A nurse has available both a 20% solution and a 4% solution of this medicine. How many milliliters of each could be mixed to prepare this 40-mL dosage?

Just a comment. If this 4% solution was made on the basis of weight, not volume, and the solute is very heavy relative to the solvent, then by volume, the percent solution might be far less than 4%. If you're a nurse, you'd better know what you are doing when mixing solutions, as suggested in math problems.

16 Word Problem #19.9

Question 25796: Milk that is 4% butterfat is mixed with milk that is 1% butterfat to obtain 18 gallons of milk that is 2% butterfat. How many gallons of each type of milk are needed? I don't even know where to start, PLEASE HELP!!!!!

Again, just a comment. This is where to start: Start with totals and their parts and the conserved quantities in this 'before and after' process.

17 Conclusion

When you encounter a mixture problem to solve, don't run off half-cocked, desperately looking for connections from the unknowns back to the given information. You can do that later. Instead, use the Scheme method of problem-solving word problems to find that system of equations that can then be solved for the unknowns you want.