Word Problems 18: Mixed-Rate Problems #15

P. Reany

November 14, 2025

Abstract

In this algebra word problem note, we use the Scheme to solve our fifteenth attempt at what I refer to as a ‘mixed-rate problem’.

1 Introduction

Some more tricky problems this time.

2 Word Problem #18.1

This is a word problem I made up, but probably someone else did also: I have long been curious what is really meant by 2% milk. According to one online source, it means that the milk fat content of 2% milk is 2% by weight. According to another source, whole milk is approximately 3.5% milk fat. Therefore, whole milk is about 96.5% skim milk. A few years ago I read on a 2% milk label that the milk inside the container was 35% less fat than whole milk. So, given just the information on that 2% milk label (and assuming it’s true), what is the fat content of whole milk?

Right off, I’m going to make a possibly controversial simplifying assumption, which I hope will not introduce a big inaccuracy into the solution, by treating percentages by weight as percentages by volume.

3 Solution 18.1.1: Conceptualizing the Problem

This bring me to one of the tricks I have learned to employ in my Scheme: If you’re given an unfamiliar situation, try to refashion it into a familiar pattern. Now, let’s conceptualize how 2% milk is made. Put simply, starting, say, with 1 gallon of whole milk, we remove a certain amount of milk fat (y gallons) until what is left is 2% milk (x gallons). If we model this as a before-and-after process, we’ll have

$$\text{(whole milk)} - \text{(some milk fat)} \rightarrow \text{(2\% milk)}\,. \tag{1}$$

But I'd prefer to visualize this process going the other way, reconstituting the whole milk, to allow me to represent the problem visually in a very familiar way, depicted in Figure 1. By the way, we lose no generality by choosing the whole milk to have a volume of 1 gallon.

In Figure 1, we see the constitutive relation between y and a, which was given to us by the claim that 2% milk is 35% less fat than whole milk. Now, our symbol for the percentage of milk fat in whole milk is a. Therefore, the amount of milk fat in one gallon of whole milk is

$$\left(\frac{a}{100}\frac{\text{gallons milk fat}}{\text{gallon whole milk}}\right)(1 \text{ gallon whole milk}) = \frac{a}{100}[\text{gallon whole milk}]\,. \quad (2)$$

But we are told that y is 35% of that, from which we get the constitutive equation

$$y = (0.35)\left(\frac{a}{100}\right)\,. \quad (3)$$

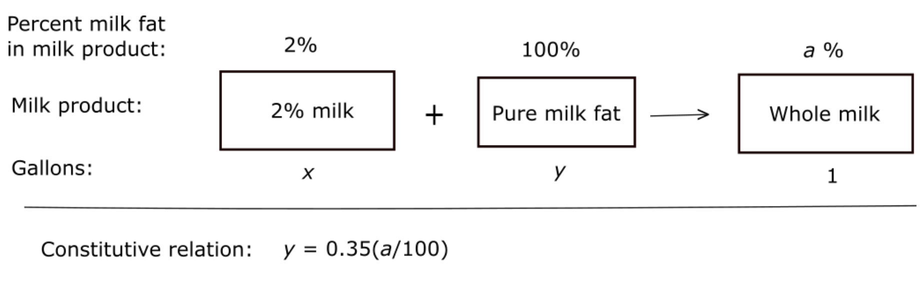

Figure 1. We can solve for the unknown value a% by reconstructing the original whole milk by putting back into it the milk fat we removed to produce 2% milk.

4 Solution 18.1.2: Solving the Problem

We can now write down the total is the sum of its parts, both before and after mixing component milk products. Balancing on milk fat, we get

$$0.02(x) + 1.00(y) = \frac{a}{100}(1)\,. \quad (4a)$$

Including the conservation of overall volumes, we get

$$x + y = 1\,. \quad (4b)$$

The solution to (3), (4a), and (4b) is (according to Wolframalpha.com)

$$a \approx 3.04414, \qquad x \approx 0.989346, \qquad y \approx 0.0106545\,. \quad (5)$$

Thus, the fat content of the whole milk from which the 2% milk came from (bearing the label I derived my information from) was a little more than 3% milk fat by volume.

5 Word Problem #18.2

Question: Ratio of gold to silver in a crown:[1]

A royal crown is an alloy of gold and solver. The crown weighs 3000 grams and has a volume of 200cc. If the density of gold is 20 grams/cc and of silver is 10 grams/cc, what is the ratio of gold to silver by volume in the crown?

6 Solution 18.2.1: Conceptualizing the Problem

Another easy problem once we identify the totals and their parts. When forming an alloy of two metals, both volumes and weights of the parts are preserved.

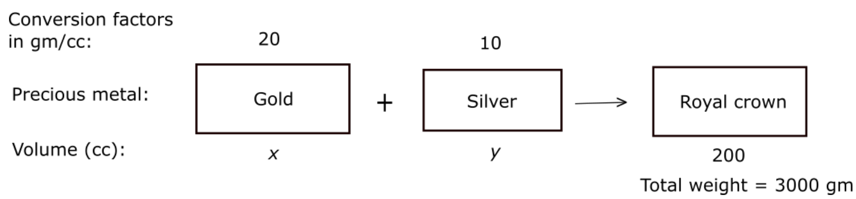

Figure 2. You can't fool this algebraist. Both weights and volumes must be conserved in forming this royal alloy.

Total volume is conserved in forming this alloy:

$$x + y = 200\,. \tag{6a}$$

And total weight is conserved:

$$20x + 10y = 3000\,. \tag{6b}$$

This system has solution $x = 100$ and $y = 100$. Therefore, the constituent metals are in ratio 1 : 1, gold to silver by volume. Lastly, since gold is twice as dense as silver, the ratio of gold to silver by weight is 2 : 1.

7 Word Problem #18.3

How much water must be added to 14 oz of a 20% alcohol solution to obtain a 7% alcohol solution? [2]

Note: The author of this problem uses something called the *bucket method* to graphically represent quantities, similar to what is represented in the Scheme presented in this series of notes on mixed-rate problems.

[1] Found at http://www2.math.umd.edu/∼jnd/Algebraic_word_problems.pdf.
[2] Found at https://www.mgccc.edu/learning_lab/math/alg/howtomix.pdf

8 Solution 18.3.1: Conceptualizing the Problem

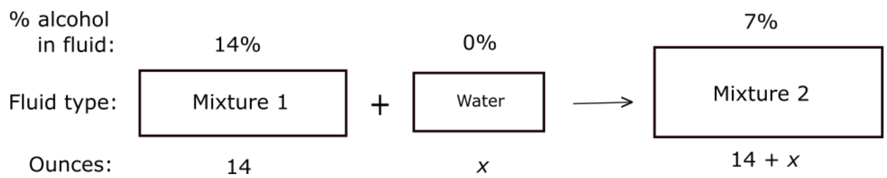

Figure 3. Standard setup for calculation: We've already shown the conservation of overall volumes in ounces.

We need just one equation, which we'll get from balancing on alcohol (the ounces, that is) on both sides:

$$(.14)x + (0)x = (.07)(14 + x)\,. \tag{7}$$

which has solution $x = 14$.

9 Word Problem #18.4

In a pen at Old MacDonald's farm there are some sheep and some geese. There is a total of 115 animals, and there are 424 legs. How many sheep and how many geese are there? (97, 18)[3]

10 Solution 18.4.1: Conceptualizing the Problem

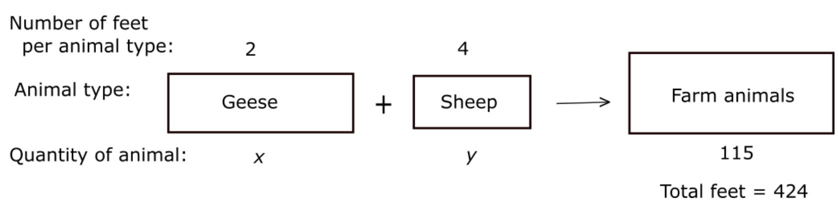

Figure 4. Standard and nonstandard information given to us.

This is a 'standard' mixed-rate problem. Let x be the number of geese and y be the number of sheep. The standard information given to us is in the form of a 'total equal to the sum of its parts', beginning with the numbers of geese and sheep:

$$x + y = 115\,. \tag{8a}$$

[3]Found at math.unm.edu/sites/default/files/files/core-courses/4.95 Mixture...

Then we have to balance on the number of animal feet:

$$(2)x + (4)y = 424\,. \tag{8b}$$

These two equations have the solution $x = 18$ and $y = 97$.

11 Conclusion

Coins in a jar, alcohol content of mixtures, feet on farm animals, words of two different point sizes on a page, volumes of two different sized jugs – they're all conceptually the same problem, just with some numeric values changed.