Word Problems 11: Mixed-Rate Problems #8

P. Reany

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Abstract

In this algebra word problem note, we use the Scheme to solve our eighth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

In this note we encounter two delightful complications to mixed-rate problems: percentages and inverse rates. We've seen them before and we deal with them again.

2 Word Problem #11.1

Let S be the subpopulation of a county of all members of age 25 years old and older. Calculate the percent of people in S that have a college degree if

1. The percent of males in S with college degrees is 26%, and the percent of females in S with college degrees is 16%.
2. The percentage females in S is 55%. (Hence, the percentage males in S is 45)

3 Solution 11.1.1: Conceptualizing the problem

Let m be the number of males in S, and let f be the number of females in S. The cardinality of S is $T = m + f$.

In the figure below, we have a standard 'total is the sum of its parts' graphic.

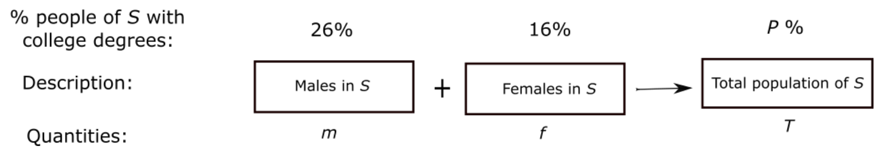

Figure 1. Males and females of S, and the percentages of each with college degrees.

First, let's be clear what we are asked to solve for. We need the percent of the people in S that have a college degree. As always, the percent P is the part over the whole times 100%. One number we will need is the total number of people in S who have college degrees, and that is solved as the sum of its parts, given from the figure:

$$\text{no. of people in } S \text{ with college degrees} = .26m + .16f\,. \tag{1}$$

We'll also need to convert m and f as functions of T. We were told that 55% of S are females, so we can conclude that $f = .55T$. And also that $m = .45T$

Now, on to the percentage we need.

$$\begin{aligned} P &= \frac{\text{no. people in } S \text{ with degrees}}{\text{no. people in } S} \times 100\% \\ &= \frac{.26m + .16f}{T} \times 100\% \\ &= \frac{.26(.45T) + .16(.55T)}{T} \times 100\% \\ &= [.26(.45) + .16(.55)] \times 100\% \\ &= 20.5\%\,. \end{aligned} \tag{2}$$

4 Word Problem #11.2

Question 22466 (modified)[1]: An apprentice A takes three hours longer to do a certain job than his mentor M. Working together, they can do the job in two hours. What are their individual rates?

5 Solution 11.2.1: Conceptualizing the problem

Let's begin without a figure this time.

We'll represent the rates at which the mentor and the assistant work the job as, respectively, R_M and R_A. These have units of job/hour, but some of the information given to us forces us to compare their hours/job, requiring the

[1]Found at https://www.algebra.com/algebra.

multiplicative inverses of R_M and R_A. So, let the time it takes M to do the job be T hours. Then, $R_M^{-1} = T$ hours/job and $R_A^{-1} = (T+3)$ hours/job. Therefore, $R_M = 1$ job/(T hours) and $R_A = 1$ job/[(T + 3) hours]

For our second equation, we take the total of one job being done by the mentor and apprentice, working for same quantity of time, 2 hours:

$$1 = \frac{1}{T}\,2 + \frac{1}{T+3}\,2\,, \tag{3}$$

with units suppressed, as usual. This equation has solution $T = 3$ hours. Then for $T+3$ we get 6 hours. We're almost done. $R_M = 1$ job/(3 hours) and $R_A = 1$ job/(6 hours).

6 Word Problem #11.3

Question 22466:[2] Working together, A and B can do a job in 4 days. Working alone, A takes twice as long as B to do the job. Find their individual rates.

7 Solution 11.3.1: Conceptualizing the problem

This problem is similar to the last one. We won't need a figure this time, either.

The rates at which A and B work the job are, respectively, R_A and R_B. Again, these have units of job/hour, but, again, some of the information given to us forces us to compare their hours/job, requiring the multiplicative inverses of R_M and R_A. So, let the time it takes B to do the job be T hours. Then, $R_B^{-1} = T$ hours/job and $R_A^{-1} = 2T$ hours/job. Therefore, $R_A = 1$ job/(2T hours) and $R_A = 1$ job/(T hours).

For our second equation, we take the total of one job being done by A and B, working for same quantity of time, namely, 4 days:

$$1 = \frac{1}{2T}\,4 + \frac{1}{T}\,4\,, \tag{4}$$

with units suppressed, as usual. Solving this for T we get 6 days. Then, $2T = 12$ days. We're almost done. $R_A = 1$ job/(12 days) and $R_B = 1$ job/(6 days).

8 Word Problem #11.4

Two vessels A and B have mixtures of milk and water. A has them in ratio 5:2 and B has them in ratio 8:7. The volume of vessel A is 2 gallons, and the volume of vessel B is 3 gallons. If the contents of A and B are mixed together, what will be the milk-to-water ratio of this mixture?

[2] Found at https://www.algebra.com/algebra.

9 Solution 11.4.1: Conceptualizing the problem

We've seen problems similar to this one before. The way to solve this problem is to move from information given as ratios to information in the form of fractions and then move back into the realm of ratios. It can be a bit tricky if you are new at it.

We need to find two positive integers x, y to present the final ratio as $x : y$. The problem is that we have only one remaining independent equation we can get out of Figure 2, which is setup to balance for milk. In the process of balancing, we end up with the faction $\frac{x}{x+y}$. The question is: How do we use that to get $x : y$? We'll see.

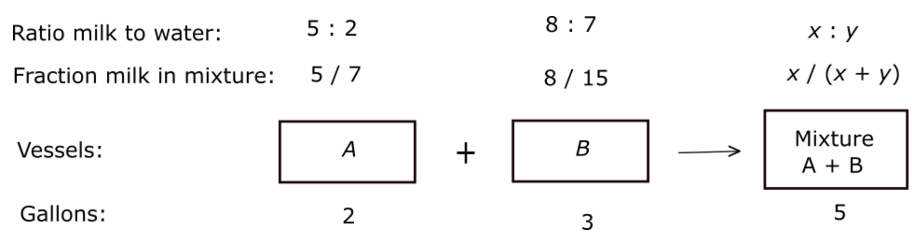

Figure 2. We're optimistically expecting to find two relatively prime positive integers x, y such that we can write the final answer as the ratio $x : y$.

10 Solution Part 11.4.2: Solving the problem

From the conservation of milk, we get that

$$\frac{5}{7} \cdot 2 + \frac{8}{15} \cdot 3 = \frac{x}{x+y} \cdot 5 \,. \tag{5}$$

Let's now employ a trick we've used before. Let $\lambda = x/y$ and rewrite $\frac{x}{x+y}$ as $\frac{\lambda}{\lambda+1}$. Now, we can rewrite (5) as

$$(\frac{10}{7} + \frac{24}{15})(\lambda + 1) = 5\lambda \,. \tag{6}$$

Wolframalpha.com gives the solution to λ as $\frac{106}{69}$. Therefore, $x : y :: 106 : 69$.