Word Problems 10: Mixed-Rate Problems #7

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Abstract

In this algebra word problem note, we use the Scheme to solve our seventh attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

At this point in the series, I think the reader who has followed the notes this far, is ready to forego some accompanying figures, as they can be repetitive from prior problems.

2 Word Problem #10.1

John can do a work in 24 days and Ethan can do the same work in 40 days. This time, John starts alone and works for 12 days before Ethan starts and then John stops. Ethan continues the job until the work is 75% finished. How much time T did Ethan work?

3 Solution 10.1.1: Conceptualizing the Problem

Let's try to solve this without the aid of a figure. But we do start with the familiar equation for the job being done by two 'machines' cooperating on the project.

$$.75(\text{job}) = (\text{PJDB John}) + (\text{PJDB Ethan})\,, \tag{1}$$

where, as before, 'PJBD' means 'the part of the job done by'. Then,

$$.75(\text{job}) = (\frac{1}{24}\text{ job/day})(12\text{ days}) + (\frac{1}{40}\text{ job/day})(T)\,. \tag{2}$$

To solve this equation for T, convert .75 to the fraction 3/4, and T is easily found to be 10 days.

4 Word Problem #10.2

Two copiers are used to do a job. Copier A works for 45 minutes, and copier B works for 30 minutes. If copier A produces twice as many copies in 5 minutes as copier B makes in 15 minutes, how fast does copier A work?

5 Solution 10.2.1: Conceptualizing the Problem

Let's solve this one as well without a figure. But we do start with the familiar equation for the job being done by two machines cooperating on the project.

$$1(\text{job}) = (\text{PJDB A}) + (\text{PJDB B})\,. \tag{3}$$

Then,

$$1[\text{job}] = R_A T_A + R_B T_B\,, \tag{4}$$

where $T_A = 45$ minutes and $T_B = 30$ minutes.

We have been asked to find R_A (which has units job/minutes), and we can do this with the help of the information connecting R_A and R_B. Let's work this out now. Let N be the number of copies that A can make in 5 minutes. We don't really care what N is, but we can write the following

$$N = R_A(5) = 2R_B(15)\,, \tag{5}$$

which comes right out of the given information. Solving this for R_B, we get $R_B = R_A/6$. And plugging this into (4), we get

$$1 = R_A(45) + \frac{R_A}{6}(30)\,, \tag{6}$$

and solving for R_A we get 1/50, meaning one job per 50 minutes.

6 Word Problem #10.3

A footwear store sells only shoes and boots. The price of all pairs of shoes are the same, namely R_S, and the price of all pairs of boots are the same, namely R_B. Determine the price per pair of shoes and boots if, on Monday, the store sold 22 pairs of shoes and 16 pairs of boots for a total of $650, and on Tuesday it sold 8 pairs of shoes and 32 pairs of boots for a total of $760.

7 Solution 10.3.1: Conceptualizing the Problem

This time, the given information in the problem is sufficiently novel for us to include a figure.

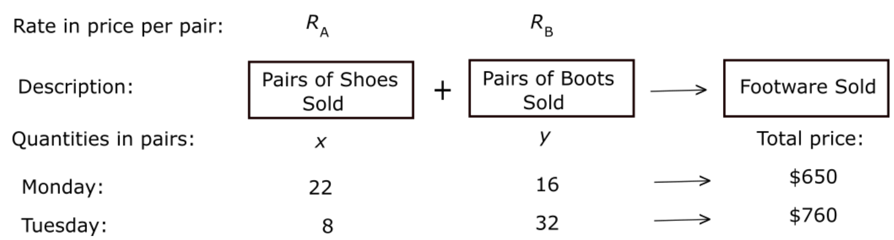

Figure 1. This graphic shows us the number of pairs of shoes and boots sold on two different days, and the net income from both of them. From this we can calculate the price per pair of shoes R_S and pair of boots R_B.

We can think of this problem as having supplied us with two equations of the form 'total is the sum of its parts' — one equation for the sales on Monday, and the other for the sales on Tuesday.

We have two unknowns to solve for, so we need two coupled equations to solve for them.

$$\text{Monday Sales:} \qquad R_S(22) + R_B(16) = 650\,, \tag{7a}$$

$$\text{Tuesday Sales:} \qquad R_S(8) + R_B(32) = 760\,. \tag{7b}$$

The solution to this system is $R_S = \$15$ per pair of shoes and $R_B = \$20$ per pair of boots.

8 Word Problem #10.4

Question 200033:[1] Soybean meal is 14% protein, corn meal is 7% protein. How many pounds of each should be mixed together to get 280 lb mixture at 13% protein?

9 Solution 10.4.1: Conceptualizing the Problem

This is a standard 'total is the sum of its parts' problem. Both overall pounds and pounds of protein in both constituent meal types is conserved in the resulting mixture.

[1]Found at https://www.algebra.com/algebra.

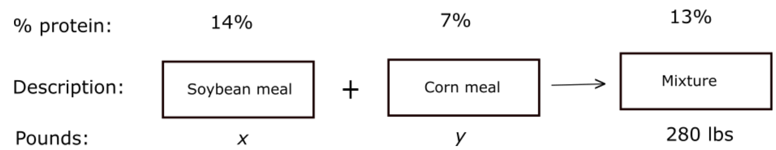

Figure 2. Protein is a physical constituent of the two kinds of meal and it will be conserved upon mixing any quantities of the two.

$$\text{Conservation of overall pounds:} \quad x + y = 280\,, \tag{8a}$$
$$\text{Conservation of protein:} \quad .14x + .07y = .13(280)\,. \tag{8b}$$

This pair of equations has solutions $x = 240$ and $y = 40$, both in pounds.

10 Conclusion

The problems in this note are of moderate difficulty. But we still have a lot more word problems to go.