A Simple Fixed-Point Theorem; The Art of the Slam Dunk

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Abstract

In this paper we learn to apply heuristics to solve a simple fixed-point theorem.

1 Introduction

I claim that this problem is a 'simple' version of the collection of fixed-point theorems as there are more difficult versions in higher mathematics than this one in calculus. Let $\mathbf{I}$ be the real interval $[0, 1]$. Then,

> If f is a continuous map of $\mathbf{I}$ to itself, then there exists a real number $c \in \mathbf{I}$ such that $f(c) = c$. That is, the map f fixes c.

Now, I suppose it must occur useful to the reader to try a proof by contradiction, and reasonably so. However, our method promoted here is a more pro forma means to accomplish that end.

Our methodology of heuristics for problem solving includes:

Heuristic 1: Solve a simpler, related problem, perhaps by introducing extra information (the Logic Splitter can accomplish this).

Heuristic 2: Be aware of all your previously proved theorems (PPTs) you can draw on. In the case of this theorem, since we need to prove the existence of a point, we should look to PPTs that also prove the existence of a point.

Heuristic 3: Introduce a Slam-Dunk proposition P that you can stick into a logic splitter.

Heuristic 4: Focus on objects in the statement of the theorems that are special or distinguished. (In our case, those are the boundary points.)

The previously proved theorem that we will draw on in this proof is the Intermediate Value Theorem (IVT):

Let $g(x)$ be a continuous function on the interval $[a, b]$, where $g(a) \neq g(b)$, then $g(x)$ takes on all values between $g(a)$ and $g(b)$.

But what is this notion of a 'Slam Dunk'? Colloquially put, a slam dunk is some action that is forcefully executed, though with little apparent effort and with great apparent effectiveness.

Let's let an Instructor and Student demonstrate it for us:
Instructor: How you doing on the fixed-point theorem I told you about?
Student: Oh, yeah! I got that one solved.
Instructor: Really? So soon?
Student: Sure.
Instructor: Well, let's have it, then.
Student: Either $f(0) = 0$ or $f(1) = 1$. At least one point is fixed! Slam dunk!
Instructor: Wait a minute. How do you know that $f(0) = 0$?
Student: I don't, but then $f(1) = 1$. Slam dunk! I win!
Instructor: Not so fast! What if neither $f(0) = 0$ nor $f(1) = 1$?
Student: Oh, dear. I see your point. I'm lost, then.
Instructor: Now, don't give up so fast. That gives me an idea.
Student: What idea?
Instructor: It's going to take a flowchart to explain.

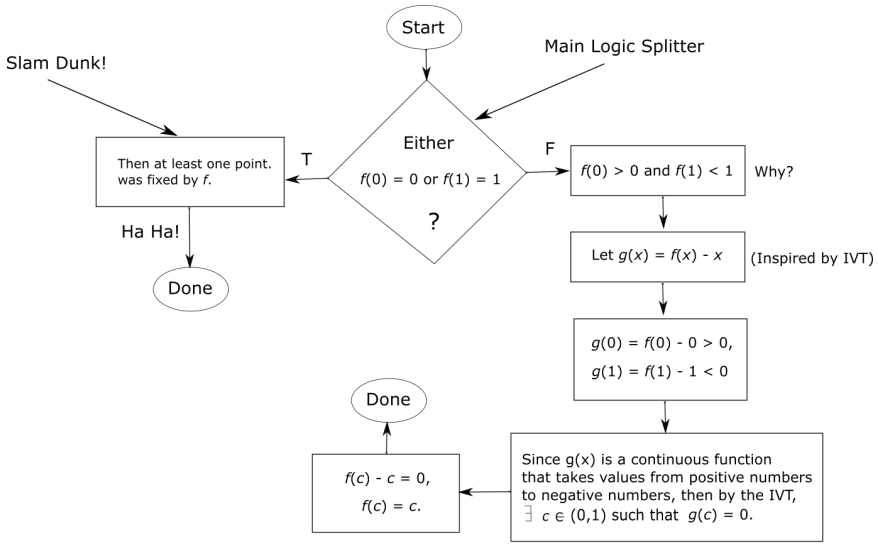

Figure 1. If your slam-dunk proposition isn't correct, then it's wrong, and knowing that it's wrong gives you useful information.