Teaching Stoichiometry as Algebraic Word Problems, Part 1

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Abstract

This paper presents the subject of stoichiometry as a collection of algebraic methods of solving chemistry word problems. First we learn how to solve ordinary word problems, or you can jump ahead to Problem 9.

1 Introduction

Stoichiometry is a basic topic of chemistry, concerned with solving for certain quantities of products and/or reactants in a balanced chemical equation, given knowledge of other quantities in the equation. Such quantities of interests are typically moles, grams, and/or liters of particular substances.

Author's Admission: I'm still in need of much more learning myself on the rudiments of chemistry, so please bear with my inevitable naive mistakes on chemistry that will likely occur from time to time in this paper.

It is not the purpose of this paper to teach all relevant aspects of chemistry needed to understand stoichiometry, such as Avogadro's Law, Avogadro's number, or the meanings of terms such as mole, isotopes, atomic mass units, molar mass, products, reactants, etc. Therefore, prior to reading this paper, the reader should already know the basics of chemical theory needed as a foundation to this topic, although, I will touch briefly on such matters as the paper progresses.

For an example of a simple stoichiometry problem, if one wants to make a certain amount of ammonia (NH_3), say 200 liters, by the Haber Process:

$$N_2 + 3H_2 = 2NH_3\,, \tag{1}$$

how many liters of nitrogen gas (N_2) at Standard Temperature and Pressure (STP) would be required?

In the nomenclature of *Scheme* — the name I gave to the methods of algebra problem solving I developed over the years[1] — Eq. (1) is referred to as a 'before-and-after' type problem. Every 'before-and-after' type problem has something

[1]See Appendix C for a brief history of the *Scheme* system.

conserved in the process, and it's this conserved quantity (or quantities) that is (or are) the basis of an algebraic equation (or system of equations) that must be solved algebraically. Examples of conserved quantities in (1) are overall mass and the mass of each particular element. Another conserved quantity is the molar amount (or individual amount) of each element in the reaction. It's on the basis of this conservation principle that the unbalanced chemical equation can be balanced in the first place.

In stoichiometric Scheme, a typical way to diagram the reaction given in (1), could be as in Figure 0 (which I'll refer to as a *Stoich diagram* for short):

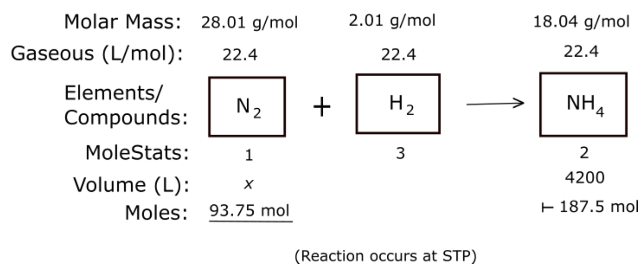

Figure 0. Diagram of the Haber Process, revealing an orderly system for keeping track of relevant information. In stoichiometric 'bookkeeping' in Scheme, we see the relevant data placed in column form. Quantities in the same column are usually related to each other by multiplication or division, whereas, quantities in the same row are often related to each other by conservation rules or by mole proportions.

The first thing I want to say about Figure 0 is a comment on the general layout of the data placed in the diagram. To begin with, typically, I place rates above the boxes and simple quantities below the boxes (following the habit I formed in solving algebra word problems). Immediately below the boxes I place the coefficients of the balanced chemical equation under investigation. I refer to this line of coefficents as the *MoleStats line.* Warning: These molestats numbers are *not* true quantities, per se, but, rather, represent mole proportions.

The second thing to say about a Scheme stoich diagram is that it contains information of two broad types: *nonderived* and *derived.* The nonderived information is in three subtypes: Given, tabular, or physical law. The given information is obviously specifically given in the problem statement. The tabular information comes from look-up tables, such as the molar masses of elements or compounds (in this paper, see Appendix A)[2]. The physical law information can also be looked up, such as the volume of a mole of an ideal gas at STP.

The derived information comes in three forms: 1) Values derived from information residing only in the same column. These values are usually prefaced by a turnstile $\vdash$ (see the figure). 2) Values derived using at least one piece of

[2]Yes, I know that in some educational situations, one must calculate molar massed from scratch, but in this paper, one can use the appendix if one wishes.

information from a column other than that in which it resides. These values are usually underlined (see figure). 3) Values derived from information (given in the problem statement or found somewhere). Such values are usually prefaced by a solid right arrowhead ▶ (not shown in figure).[3] Values derived from information lying outside the diagram should be explicitly calculated outside the stoich diagram, or at least alluded to, for the reader's benefit. As a final comment on these markup symbols, they play a role in stoich diagrams similar to comments placed in computer code, i.e., they help to clarify, at a glace, the origin of the data.

Referring to the diagram, the first and easiest calculation to do is to derive the moles of NH_3:

$$\text{moles NH}_3 = \frac{4200 \text{ L}}{22.4 \text{ L}\cdot \text{mol}^{-1}} = 187.5 \text{ mol}\,, \tag{2}$$

where all the information needed to perform this calculation came from the same column as the value that was derived — hence the turnstile indicator.

Now, the way to use this new mole information toward the goal of finding the liters of N_2 gas, is to use what I refer to as the *mole proportion*[4] between the respective columns (*column hopping*): *The ratio of actual moles of substances from two columns is equal to the ratio of their respective Molestats numbers.* This ratio is sometimes referred to as the *stoichimetric ratio.* Hence,

$$\frac{\text{moles N}_2}{\text{moles NH}_3} = \frac{1}{2}\,. \tag{3}$$

On solving this for the moles of N_2 (with moles NH_3 = 187.5 mol), we get 93.75 mol. And now we're ready to solve for x, the number of liters of N_2 gas we need to solve for.

$$x = 93.75 \text{ mol} \times 22.4\,\text{L}\cdot \text{mol}^{-1} = 2100\,\text{L}\,. \tag{4}$$

I refer to the use of information in one column to be used, directly or indirectly, for making calculations in another column as *column hopping.* In this last problem, we were column hopping between columns 1 and 3. Various authors make diagrams to reveal this notion of column hopping. I'll refer the reader to just one:

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http://www.oneonta.edu/faculty/viningwj/Chem111/
          Chapter_03_bv.pdf     (p. 3-28)
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However, note that there is no equation to solve in this version of column hopping, just a set of consecutive conversion factors to apply. One of the objectives of this paper is to reveal that those modifications of one expression by a number of conversion factors to derive a final result, always begins with an equation.

[3]For example, in a gravimetric analysis in which a precipitate is collected, the given information could be the weight of the precipitate on a collection sheet and the weight of the sheet alone. But the value we need to put into the stoich diagram is the difference of these two numbers.

[4]A *proportion* is defined as the stated equality of two ratios.

Now, if a fairly good high-school algebra student were to open a chemistry textbook for the first time and thumb through the section on stoichiometry, he or she would not be too far off to conclude that the subject apparently has no need for algebraic equations, but rather relies on a trick of multiplying some given quantity by a number of conversion factors to derive a final result, one of those factors often being the stoichimetric ratio.

If this baffled student were to read on and learn that the basis of stoichiometry is the conservation of mass and moles of substances in a comparison of before-and-after states of what is referred to as a *chemical reaction*, he or she might wonder, rightly, if this book presentation hasn't actually hidden the conservation equations that underlie the computations.

One purpose of this paper is to reveal these 'hidden' conservation equations and reveal them as mere algebraic equations similar to those found in algebra word problems that the student is, or rather, should, already familiar with.

For a warmup to the subject, I'll present a number of word problems done in *Scheme* that will foreshadow those in typical stoichiometry problems. But if you're impatient to get to real stoichiometry problems, you can skip down to Section 10, and go from there.

2 Word Problem 1: The Coin Problem

► A jar containing nickels and dimes has $1.05 worth of coins in it. If the jar contains exactly 16 coins, how many are nickels and how many are dimes?

Now, as it strands, this problem is ambiguous. What we need to know is if there are *only* nickels and dimes in the jar. By application of the Zeroth Rule of Problem Solving,[5] it is reasonable to assume that there are only nickels and dimes in the jar.

SOLUTION:

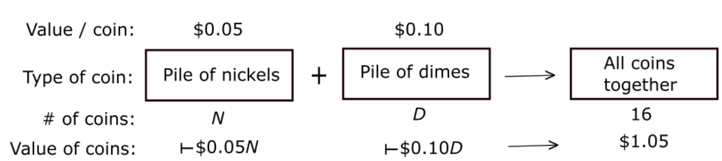

Figure 1. This graphic represents our imagined sorting of all the coins into a pile of nickels and a pile of dimes, leaving invariant the number of each.

Conceptualizing the problem. Generally speaking, unless there is an obvious reason *not* to start our solution by looking for a total, let's do so.[6]

[5] This rule states that one must make any assumption necessary to solve the problem in a reasonable amount of time with a reasonable amount of effort. It is explained more in the series of papers I made on Scheme. Basically, one employs it to resolve ambiguities or to supply missing information.

[6] When searching for 'parts', we need to find enough of them to added up to the total we

There are two obvious totals in the given information. The first is the total money in coins, being \$1.05. The second is the total number of coins in the jar. Now, this is where the assumption that there are only nickels and dimes in the jar comes in handy. You see, our procedure is to find a total, discover all of its parts, add those parts together, and set that sum equal to the total. Since we assume that the parts exist only as nickels and dimes, we begin our formulation of this equation in the simple, easy-to-understand form of

$$\text{(dollar value in nickels)} + \text{(dollar value in dimes)} = \$1.05\,. \tag{5}$$

As part of our conceptualization of this process of sorting the coins by type, we we can abstract this invariant process in Figure 1.

Now, before we make our first step-wise refinement of Eq. (5), let's ask a more general question in preparation. What does it mean to calculate the dollar value of a pile of coins of a single type? It means to count the number of coins and then multiply this number by the dollar value of each coin:

$$\text{(value of a single coin)(\#coins)} = \text{value of pile of coins}\,, \tag{6}$$

where we have placed the conversion factor to the left of the number of coins, which is customary but not necessary.[7]

Time out, please!
A *conversion factor* is a rate of change; specifically, the rate of change of things in one unit (in the numerator) into things into some other unit (in the denominator), and vice versa. Perhaps you think that it's more proper to restrict our notion of a conversion factor to converting between things of 'like nature', such as in the case of converting between inches to feet or yards to meters — all dealing with lengths or distances in this particular example. But this is an unhelpful and unnecessary restriction. What we really want is to form a conceptual basis for solving algebra problems in which the least number of primitive notions conceivable can cover the most number of particular cases.

One of my favorite examples of both 'totals being the sum of their parts' and the use of conversion factors rolled into the familiar example of what we owe on our grocery purchases: the total cost of groceries. To simplify matters, we'll assume we're buying at the grocery store two types of untaxed groceries and paying with cash. Suppose we are buying four of one kind of apple at \$0.50/apple and three cans of peas at \$1.14/can. Again, we begin with a 'total' equation:

$$\text{(total grocery bill in \$)} = \text{(\$ cost of apples)} + \text{(\$ cost of cans of peas)}\,. \tag{7a}$$

seek. However, we need to be sure that the parts are mutually exclusive (they don't intersect) so that we don't exceed the total. This is what is meant by the expression 'mutually exclusive and collectively exhaustive'.

[7] In fact, in stoichiometry (chemistry) the practice is to successively pile on conversion factors on the right.

A stepwise refinement of this last equation, gives

$$\begin{aligned}\text{total \$ grocery bill} &= \frac{\$0.50}{\text{apple}}(4 \text{ apples}) + \frac{\$1.14}{\text{can of peas}}(3 \text{ cans})\\ &= \$2.00 + \$3.42\,, \end{aligned} \tag{7b}$$

where each of these last two terms is called a *subtotal.* And the total cost of our groceries is \$5.42.

At the conceptual level, what is going on here? The conversion factors are telling us how much (many) goods we can take from the store converted into how much cash we must leave at the register.

Returning to our coin problem, the value of a single unspecified type coin is $\frac{\$\text{X.YZ}}{1\text{coin}} = \frac{\$\text{X.YZ}}{\text{coin}}$, dropping the superfluous 1 in the denominator. Now, just to be a bit exotic, let's say the coin in question is a \$20 gold piece, and we have twenty of them. Then Eq. (6) becomes

$$\left(\frac{\$20.00}{\text{coin}}\right)(20\,\text{coins}) = \$400.00\,. \tag{8a}$$

In the language of 'units' in algebra, we say that in the above equation the coin unit has 'cancelled out'. We could have made this more explicit by writing

$$\left(\frac{\$20.00}{\cancel{\text{coin}}}\right)(20\,\cancel{\text{coins}}) = \$400.00\,. \tag{8b}$$

Time in. (Thanks for your patience!)

Our first **step-wise refinement** on Eq. (5) yields

$$(\text{\$ value of a nickel})(\#\text{nickels}) + (\text{\$ value of a dime})(\#\text{dimes}) = \$1.05\,. \tag{9}$$

We still have not yet introduced any variables in this algebra problem. We could have at the start, and it wouldn't have hurt, but it wouldn't have helped much either. Let's introduce them now, setting $D = \#\text{dimes}$ and $N = \#\text{nickels}$. Then, for our next **step-wise refinement** we get

$$\left(\frac{\$0.05}{\text{nickel}}\right)(N \text{ nickels}) + \left(\frac{\$0.10}{\text{dime}}\right)(D \text{ dimes}) = \$1.05\,. \tag{10}$$

Thus, we have two unknowns but only one equation. So, we need one more equation to be able to solve for the two unknowns. Now, what I am about to write may seem terribly pedantic, but if we were writing these equations in a computer language with strong typing requirements, we would have to pay very close attention to the units of our subtotals. We actually did that properly when we considered the units in the subtotals of the 'value' equations above. But now it's time to write down the total coins equation:

$$(\#\text{ nickel coins}) + (\#\text{ dime coins}) = \text{total coins} = 16 \text{ coins}\,. \tag{11}$$

In other words, coins + coins = coins.

Fortunately, I don't intend to be this pedantic about units in future word problems, but I wanted to be very clear about the meaning of 'adding subtotals to get a total' once. Now, I'll strip the equations of *all* units and write

$$1.05 = 0.05N + 0.10D\,, \tag{12a}$$

$$16 = N + D\,. \tag{12b}$$

This system has the unique solution $N = 11$ and $D = 5$. Thus, there are 11 nickels and 5 dimes in the jar.

3 Word Problem 2

This problem was adapted from the online problem found at

`https://www.tcyonline.com/discuss/que/38331/in-what-ratio-water-added-liquid-costing-rs12-per-litre-so-make-profit-25sel`

► In what ratio should water be added to a liquid costing \$12 per liter so as to make a profit of 25% by selling the diluted liquid at \$13.75 per liter?

SOLUTION:

Conceptualizing the problem. We'll worry about the ratio of 'water added to liquid' after we have calculated how much water should be added to the starting liquid, which we'll set at 1 liter. We lose no generality by doing this.

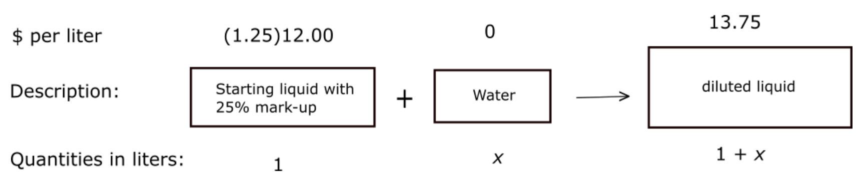

Figure 2. This graphic represents the adding of some quantity x of water to a starting liquid in a 'before and after' process. The conservation of fluid volumes has already been accounted for. We assume the water costs nothing.

But first, a word about this 25% profit. How do we deal with it? Percentage converts to a decimal, 25% to 0.25 as a multiplier. I won't go into details because I offer this only as a refresher, since the reader is presumed to be familiar with this already. (You can think of these costs as full costs or as costs per liter.)

$$\begin{aligned}(\text{retail cost}) &= (\text{base cost}) + (\text{profit})\\ &= (\text{base cost}) + (0.25)(\text{base cost})\\ &= (1.25)(\text{base cost})\end{aligned}$$

Getting the Numbers.

In the graphic in Figure 2, the conservation of volumes has already been applied. Next, we uphold the artificial 'conservation' equation of value of the liquids:

$$\text{(markedup value of original liquids)} = \text{(required value of diluted liquid)}\,. \quad (13)$$

We are told that the marked-up value to the consumer will bring in a 25% profit. In other words, the retail cost of the diluted liter of liquid is (1.25)($12.00). So, we have that the total cost of Mixture 1 and the water before they are mixed together is equal to the value of the mixture resulting from mixing them together, yielding the conservation-of-cost equation (in dollars)

$$(1.25)(12.00) + 0 \cdot x = (13.75)(1 + x)\,. \quad (14)$$

Solving this, $x = 0.090909....$ But we are asked to find the ratio of $x : 1$ (that is the ratio of volume of water added to the original volume), which is

$$0.090909 : 1 \quad \text{or (approximately)} \quad 1 : 11. \quad (15)$$

4 Word Problem 3

This problem is adapted from the webpage problem:

`https://gmatclub.com/forum/a-merchant-has-100-lbs-of-sugar-part-of-which-he-sells-at-98035.html`

► A merchant has 100 lbs of sugar, part of which (x lbs) he sells at 7% profit and the rest (y lbs) at 17% profit. The division of the whole into two parts is to be made so that the net profit is the same as 10% on each original quantity of sugar. How many pounds is each part?

SOLUTION:

Let's begin with a diagram to help us conceptualize the data.

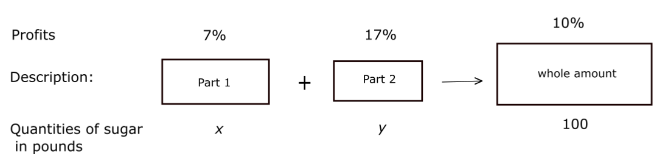

Figure 3. How to partition 100# of sugar into two parts to get 10% profit? Note: The symbol '#' is used to stand for pounds.

The net profit of 10% of the original 100# to be carried over results from the right choice of x and y, yielding the correct subtotals.

Getting the Numbers.

So, we have two equations in two unknowns, beginning with the **conservation of weight** of sugar (in pounds):[8]

$$x + y = 100\,. \tag{16}$$

And we have the **conservation of profit**:

$$(\text{profit off of } x\#) + (\text{profit off of } y\#) = (\text{profit off of } 100\#)\,. \tag{17}$$

For the next refinement, we'll convert percentages to decimals and multiply rates times quanties off Figure 3, to get

$$.07x + .17y = .10 \cdot 100 = 10.00\,. \tag{18}$$

Solving (16) and (18) together yields $x = 70\#$ and $y = 30\#$.

Follow-up:

I emphasize that in a 'before and-after process', the most likely places to look for the equations you'll need to solve the problem are in the conserved quantities in the process, such as total weights and volumes. The conservation of these quantities is guaranteed by physical law (to a high degree of approximation in most cases) and attested to by common experience, so they are straightforward to deal with. But arbitrary quantities, such as profits, are not justified by physical law or convention and must take on the logical form of *arbitrary constraints* on the system. From the psychological perspective, therefore, they take a bit more effort to get used to.

Other conserved quantities, like the dollar value of a collection of coins, are arbitrary in the sense that they're not set by physical law, yet are fixed by convention or definition before the problem is even presented.

5 Word Problem 4

This problem can be found at:

`https://m4maths.com/10925-The-milk-and-water-in-two-vessels-A-and-B-are-in-the-ratio-4-3-and-2-3-respectively.html`

or

`https://www.quora.com/The-milk-and-water-in-two-vessels....`

With respect to the latter reference, see footnote.[9]

[8] There are authors who would treat this kind of problem as essentially only a one-variable problem by using $y \to 100 - x$ to begin with (which I call *accelerated substitution*). I do this as well, sometimes. But I want the student to know how to solve more general situations first, such as would arise if we divided the original amount into three or more parts, before showing them the short cuts that work only in simpler situations.

[9] There are a number of solutions presented there for this problem, including the method of alligation.

► Two vessels A and B contain milk and water in ratios 4 : 3 and 2 : 3, respectively. In what ratio should they be added together so that their final mixture is in ratio 1 : 1?

SOLUTION:

Conceptualizing the problem

Notice in Figure 4 (below) that we used arbitrary volume units. One reason for this is that we weren't given a specific unit to work with, and the other is that we can choose any particular unit we please because in taking ratios the units will cancel, anyway.

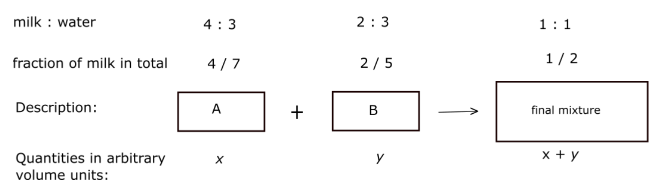

Figure 4. We need to solve for the ratio of x and y. The fractional amount of milk in a given vessel is calculated directly from the ratio given above it.

Getting the Numbers.

Now, we've already accounted for the conservation of volume in the bottom line of the diagram above. We need now only one more equation in x, y to solve for their ratios. For that, we show the conservation of milk in the process.[10]

$$(\text{milk in } A) + (\text{milk in } B) = (\text{milk in final mixture})\,. \tag{19}$$

On using the data in the above figure in (19), we get

$$\frac{4}{7}x + \frac{2}{5}y = \frac{1}{2}(x + y)\,. \tag{20}$$

The variable we need to solve for is x/y, hence, we do not need an additional equation! To efficiently solve for this, let's divide the last equation through by y, and set $\lambda = x/y$, to get

$$\frac{4}{7}\lambda + \frac{2}{5} = \frac{1}{2}(\lambda + 1)\,, \tag{21}$$

with solution $\lambda = 7/5$.[11] Therefore, $x : y :: 7 : 5$.

[10]The figure was setup to show the conservation of milk, or, alternatively, we could have employed the conservation of water to get an equation in x and y.

[11]I used WolframAlpha.com to solve for λ.

6 Word Problem 5

This problem can be found at:

`https://gmatclub.com/forum/a-can-contains-a-mixture....`

► A can contains a mixture of two liquids A and B in ratio $7:5$. After 9 liters are drawn off and replaced by 9 liters of liquid B, the ratio of A to B becomes $7:9$. How many liters of liquid A was in the can initially.

SOLUTION:

We begin by labeling the initial volume of fluid in the can as x. Since we draw off 9 liters and replace it by 9 liters, the final liquid will have x liters in it. Once we determine x, we can then solve for the initial value of A in the can. To simplify the analysis, we'll take as our 'effective' starting condition the state just after the 9 liters of fluid has been drawn off.

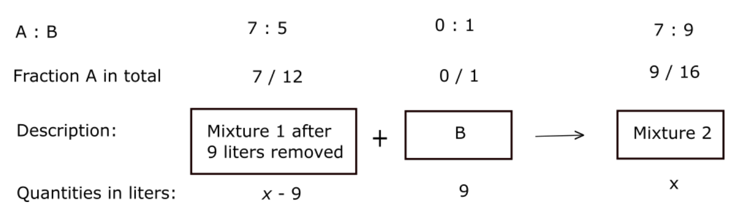

Figure 5. This graphic represents adding 9 liters of liquid B to a $x-9$ liters of a starting mixture (Mixture 1) in a 'before and after' process. 'Fraction (amount of) A in can' means $A/(A+B)$.

We continue with our usual conservation equation, this time for the volume of liquid A.

$$\text{(total A in can before adding B)} = \text{(total A in can after adding B)}\,. \tag{22}$$

Keeping in mind that no part of the nine liters added contains any bit of liquid A, we get

$$\frac{7}{12}(x-9) + 0\cdot 9 = \frac{7}{16}(x)\,. \tag{23}$$

Getting the Numbers.

Therefore the solution for x is 36 liters. And seven-twelfths of that is the initial volume of liquid A in the can was,

$$\text{inital amount of } A = \frac{7}{12}\cdot 36\,\text{liters} = 21\,\text{liters}\,. \tag{24}$$

7 Word Problem 6

► A heat-loss survey by an electrical company indicated that a wall of a house containing 40 ft^2 of glass and 60 ft^2 of plaster lost 1920 BTU of heat (in a given time period). A second wall containing 10 ft^2 of glass and 100 ft^2 of plasteer lost 1160 BTU of heat. Determine the heat lost per square foot of glass and plaster in that house. (This problem comes from *Intermediate Algebra for College Students*, 3rd Ed. [2], p. 169–171.)

SOLUTION:

Conceptualizing the problem

Let R_G be the rate of heat lost per square foot through glass, and R_P be the rate of heat lost per square foot through plaster.

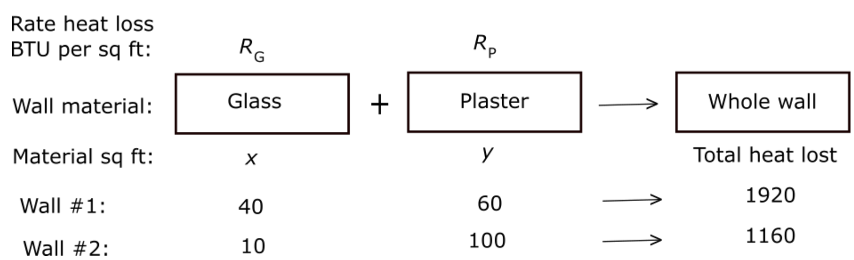

Figure 6. Heat leakage through glass and plaster.

Somehow this clever heat-loss technician is able to measure the heat lost through an entire wall. He then measures the square footage of the glass and plaster of this wall, and repeats this process for another wall, and then uses algebra to infer the heat loss through just the glass or just the plaster.

We can do this ourselves. The total heat lost for both walls is equal to the respective sums of the heat lost through their glass parts and their plaster parts:

$$\begin{aligned} 1920 &= 40x + 60y\,, \\ 1160 &= 10x + 100y\,, \end{aligned} \tag{25}$$

where $x = R_G$ and $y = R_P$. This substitution makes it easier to copy the text into a solver, which gives back $x = R_G = 36$ [BTU] and $y = R_P = 8$ [BTU].

8 Word Problem 7

This problem can be found at:

`https://m4maths.com/5846-Nine-litres-are-drawn-from-a-cask-full-of-wine-and-it-is-then-filled-with-water-Nine-litres-of.html`

► Nine liters are drawn from a tank full of wine. Then 9 liters of pure water are added to the tank and the mix is allowed to homogenize. After that, 9 more liters are drawn off and again replaced by 9 liters of pure water and allowed to homogenize. If the final wine-to-water mix is in ratio 16 : 9, how much does the full tank hold?

SOLUTION:

Conceptualizing the problem

We already did a similar problem in Word Problem 5. We can solve this problem in two steps. First step, let x be the full volume of the tank, containing pure wine = Mix 0. After replacing 9 liters of the orignal wine by 9 liters of water, we end up with Mix 1 with wine-to-water ratio $(x-9) : 9$ (of volume x) and fraction of wine-to-total mixture $(x-9)/x$ (no figure for this point in the analysis).

Second step, we draw off 9 more liters of this Mix 1 and replace that by 9 liters of pure water, resulting in the final state, Mix 2. The setup is depicted in Figure 7.

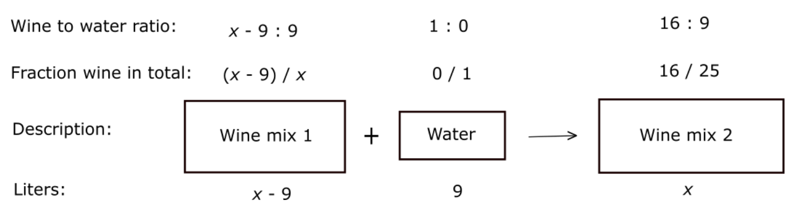

Figure 7. Shown here is the second step of this two-step process: It starts with Mix 1 (after 9 liters have been removed) and ends up with Mix 2. (Whereas, Step 1 started with Mix 0 and ended with Mix 1.)

Getting the Numbers.

The process of adding pure water to Mixture 1 (Figure 7) allows us to write down a simple conservation equation for wine:

$$\frac{x-9}{x}\,(x-9) + 0(9) = \frac{16}{25}\,(x)\,, \tag{26}$$

which has solution $x = 45$ liters.[12]

9 Word Problem 8

► A woman must control her diet. She selects milk and bagel for breakfast. How much of each should she serve in order to consume 700 calories and 28 grams of

[12]The possible root $x = 5$ is unphysical because it does not satisfy the constraint $x - 9 > 0$.

protein? Each cup of milk contains 170 calories and 8 grams of protein. Each bagel contains 138 calories and 4 grams of protein. (Problem found in [4].)

SOLUTION:

Conceptualizing the problem

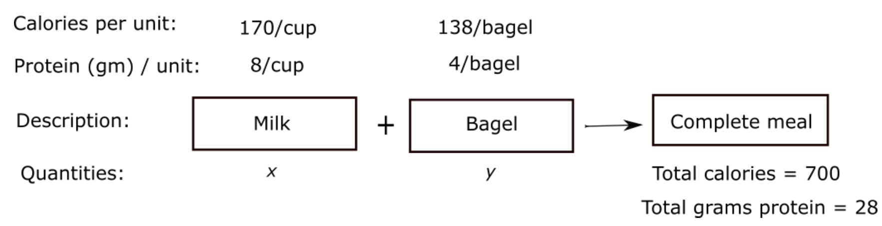

Figure 8. As usual, rates are placed above and quantities (and totals) are placed below.

Getting the Numbers.

We simply have two totals to deal with, employing the given constraints on the amounts of each. Referencing Figure 8, we have that

$$170x + 138y = 700\,, \tag{27a}$$
$$8x + 4y = 28\,, \tag{27b}$$

which has solution

$$x = \frac{133}{53} \approx 2.5 \quad \text{and} \quad y = \frac{105}{53} \approx 2\,. \tag{28}$$

That is, the meal is to consist of 2.5 cups of milk and 2 bagels.

10 Word Problem 9: The Boron Problem

► Naturally occurring Boron (molar mass of 10.81 g/mol = $10.81\,\mathrm{g{\cdot}mol^{-1}}$) is the mixture of two of its isotopes: Boron 10 (^{10}B) and Boron 11 (^{11}B), of atomic masses 10.01 g/mol and 11.01 g/mol, respectively. Find the relative abundances of ^{10}B and ^{11}B in natural Boron, expressed in percentages.

SOLUTION:

Conceptualizing the problem

First, take note that the mixing of these two isotopes to form natural Boron is a physical, not a chemical, mixing. We shall model this problem as mixing the two isotopes in the right proportions to yield naturally occurring Boron. We lose nothing by assuming one mole of natual boron to begin with.[13]

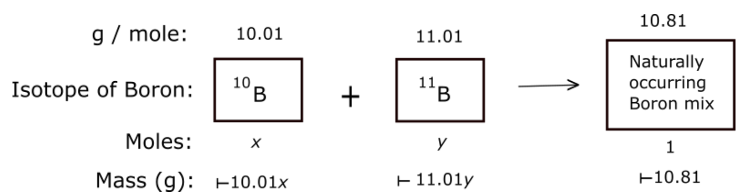

Figure 9. This graphic represents our imagined sorting of 1 mole of naturally occurring Boron into logical piles of ^{10}B and ^{11}B.

Referring to the figure above, it should be clear, after a little comparison with the coin problem at the beginning of this paper, that the two problems are very similar. In the coin problem, we had two conservation equations: one for the conservation of the total number of coins, and another for the conservation of the total value of the coins.

In this problem, we have one equation for the conservation of the total number of moles of isotopes, and another for the conservation of the total grams of the isotopes, from which we get the pair of equations to be solved simultaneously:

$$x + y = 1\,, \tag{29a}$$

$$10.01x + 11.01y = 10.81\,. \tag{29b}$$

Getting the Numbers.

Wolframalpha gives the approximate values for x and y as $x \approx 0.2$ and $y \approx 0.8$. Converting these values to percentages, we have that in naturally occurring Boron, the relative abundance of ^{10}B as about 20% and ^{11}B as about 80%.

11 Preparing for Stoichiometry

One of the first things to do in a typical stoichiometry problem is to balance an unbalanced chemical equation. We won't go into the strategies for accomplishing that in this paper. But let's do one of them now for practice.[14] Consider the unbalanced chemical equation

$$\mathrm{Li} + \mathrm{N_2} \longrightarrow \mathrm{Li_3N}\,. \tag{30}$$

[13] If this were not the case, then the relative percentages of these two isotopes of boron in natural boron would be functions of the macroscopic amount of natural boron collected, all other things being equal. But if this were true, then claiming that natural boron has the fixed molar mass of 10.81 would be untrue and/or meaningless.

[14] Sometimes a purely algebraic method of solving for the coefficients is obtainable. See Appendix B.

The end objective of balancing any chemical equation is to have, for each element, the same number of each on the left side of the equation as on the right side. Clearly, 'equation' (30) is unbalanced because there are three lithiums on the right for every one lithium on the left. The nitrogens are also unbalanced. One possible first step to balancing this is the following

$$3\text{Li} + \tfrac{1}{2}\text{N}_2 \longrightarrow \text{Li}_3\text{N}\,. \tag{31}$$

This certainly completed the task of balancing each element in one step; however, convention is (usually) to clear the equation of fractions. So, we'll multiply through by 2, to get

$$6\text{Li} + \text{N}_2 \longrightarrow 2\text{Li}_3\text{N}\,. \tag{32}$$

Notice that we did not place a 1 in front of the N_2 term on the left, since its presence is implied by convention.

Say that our problem is to determine the grams of N_2 that must be supplied to react with 41.64 grams of lithium to produce 69.66 grams of Li_3N? Let's start by forming a graphic of this process:

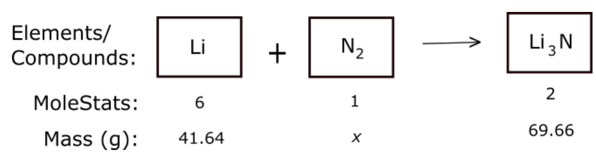

Figure 10. This graphic represents the conservation of mass when of 41.64 grams of lithium react with x grams of nitrogen to produce 69.66 grams of Li_3N.

The line labeled *MoleStats* refers to the coefficients of the terms in the balanced equation; however, we won't be using it this time, but it will play a crucial role in most problems we encounter later. Now, it may seem obvious how to proceed at this point. We merely assume that the grams are conserved in this reaction and therefore write down that

$$41.64\,\text{g} + x = 69.66\,\text{g} \tag{33}$$

and solve for x to get $x = 28.02$ grams. This is correct, but since chemistry is a science, we need more than intuition or common sense to justify this train of thought.

One of the fundamental laws of chemistry is the Law of Conservation of Mass, stated in a form fit for chemistry:

`In a closed system, the sum of all masses of the reactants in a chemical reaction is equal to the sum of all masses of the products of the reaction.`

This law justifies our calculation, but the reader should know that this kind of problem is a bit too simplistic to be seen often in stoichiometry.

The last comment I wish to make before we enter into solving for more typical stoichiometry problems is the role played by the numbers in the MoleStats line of the figures we draw.[15] Since this paper is meant to present stoichiometry as algebra, it uses the fact that the elements and/or compounds that react and produce products, do so by fixed ratios. For example, referring back to Figure 10, the ratio of lithium to Li_3N is 6 : 3. And the ratio of N_2 to Li_3N is 1 : 2. We'll see in the next problem how to form an algebraic equation out of this by finding the Fundamental Proportion to the problem (relative to any two particular terms of the balanced equation).

Thus, more important to stoichiometry, and in particular for our algebraic treatment of it, is the fundamental *Law of Fixed Molar Ratios*, stated as:[16]

`In a chemical reaction, the ratio of the coefficients of any two terms is a fixed rational number which is equal to the ratio of the moles of the respective chemical substances in the reaction.`

For example, look again at chemical equation (32). From it we can conclude that the moles of Li to the moles of N_2 is 6 : 1, which can be written in algebraic form as the proportion[17]:

$$\frac{\text{moles Li}}{\text{moles N}_2} = \frac{6}{1}\,. \tag{34a}$$

Similarly, we can conclude a similar proportion:

$$\frac{\text{moles Li}_3\text{N}}{\text{moles Li}} = \frac{2}{6}\,. \tag{34b}$$

And also:

$$\frac{\text{moles Li}_3\text{N}}{\text{moles N}_2} = \frac{2}{1}\,. \tag{34c}$$

Of course, we know that we can invert the fractions of both sides of any of the last three equations and still have a valid equation.

12 Problem 1: Kilograms to kilograms

This first problem is taken from the chemistry textbook *Chemical Principles: The Quest for Insight* ([1], p. F82):

PROBLEM:

► What mass of aluminum is needed to reduce 10.0 kg of chromium (III) oxide to produce chromium metal? The chemical equation for the reaction is

$$2\text{Al(l)} + \text{Cr}_2\text{O}_3\text{(s)} \xrightarrow{\Delta} \text{Al}_2\text{O}_3\text{(s)} + 2\text{Cr(l)}\,. \tag{35}$$

[15] I use the term 'Molestats' in reference to the statistical characteristic of 'organization and presentation of data', admittedly the weakest sense of the word *statistics*, but 'MoleStats' does have a nice sound to it.

[16] A similar notion is The Law of Definite Proportions.

[17] Remember that a proportion is the claimed equality of two ratios, and in this context, the ratio is called the *stoichiometric ratio*.

SOLUTION:

As a note to the reader: Appendix A contains a list of molar masses of various compounds for the problems in this paper, although, slight differences can occur in these molar masses depending on differences existing in various source references.

Conceptualizing the problem

First, we make a figure (below) that contains all relevant data:

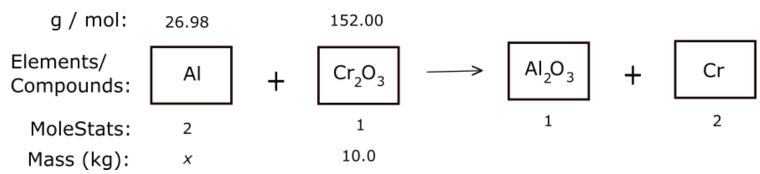

Figure 11. A kilograms-to-kilograms problem.

Next, we formulate the fundamental (mole) proportion[18] of this problem:

$$\frac{\text{moles Al}}{\text{moles } \mathrm{Cr_2O_3}} = \frac{2}{1}\,. \tag{36a}$$

But now we have to relate moles to kilograms, which is simple. Switching sides and expanding with the given information, yields:

$$\frac{2}{1} = \frac{\text{moles Al}}{\text{moles } \mathrm{Cr_2O_3}} = \frac{x[\text{kg Al}]/26.98\,\text{g/mol}}{10.0\text{ kg}[\mathrm{Cr_2O_3}]/152.00\,\text{g/mol}}\,. \tag{36b}$$

where the information in the square brackets is parenthetical.

Now, I must explain that I pulled a fast one in Eq. (36b). There is actually nothing incorrect about what I did in writing this proportion; however, your chemistry teacher may not think so. For purists, I should have written

$$\frac{2}{1} = \frac{\text{moles Al}}{\text{moles } \mathrm{Cr_2O_3}} = \frac{x[\text{kg Al}]/26.98\times 10^{-3}\,\text{kg/mol}}{10.0\text{ kg}[\mathrm{Cr_2O_3}]/152.00\times 10^{-3}\,\text{kg/mol}}\,. \tag{37}$$

This is the kind of subtlety with units that trips-up new students. Anyway, the reason (37) is just as good as (36b) is because we can go between them by merely multiplying both numerator and denominator by the same conversion factor. However, if we needed to know the moles of Cr_2O_3, given that we know there are 10.0 kg of it, then the correct units in the calculation would be

$$\text{moles } \mathrm{Cr_2O_3} = \frac{10.0\ \cancel{\text{kg}}[\mathrm{Cr_2O_3}]}{152.00\times 10^{-3}\ \cancel{\text{kg}}/\text{mol}}\,. \tag{38}$$

[18] I refer to this proportion as involving moles, but it could just as easily refer to ratios of numbers of molecules. However, the masses of compounds are usually given in terms of molar masses, not molecular masses.

Solving (36b) for x and canceling units where possible, we get

$$x = \frac{2}{1} 26.98 \frac{10.0 \text{ kg}}{152.00} = 3.55 \text{ kg}\,. \tag{39}$$

Now that we've finished our first real stoichiometry problem, what have we learned about the solution? We've learned that the solution to this problem is much the same as the solutions seen above to ordinary algebra word problems, especially those that include conversion factors and proportional reasoning.

A website that sets up the setting of the mole proportions similarly to how it's done here is at

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http://www.chembuddy.com/?left=balancing-stoichiometry
         &right=ratio-proportions
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13 Problem 2: Titration of Oxalic Acid

This second problem is taken from the same textbook *Chemical Principles: The Quest for Insight* ([1], p. F84):

PROBLEM: (Paraphrased) 25.00 mL of oxalic acid is titrated with 0.100 M NaOH(aq) until all the acid is consumed. If it required 38.00 mL of base to reach this point, what was the molarity (moles/liter) of the acid?

SOLUTION: First, the base referred to is NaOH. The chemical equation for the reaction is

$$H_2C_2O_4 + 2NaOH \longrightarrow Na_2C_2O_4 + 2H_2O \tag{40}$$

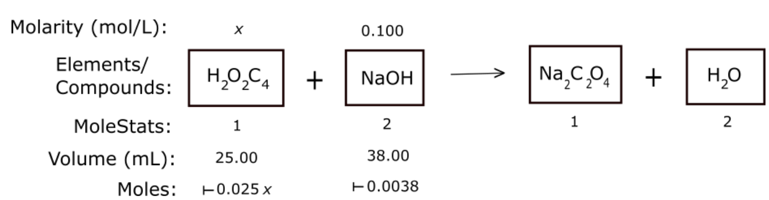

Figure 12. Oxalic acid titration by NaOH.

Next, we write down our mole proportion on columns 1 and 2:

$$\frac{1}{2} = \frac{\text{moles } H_2C_2O_4}{\text{moles NaOH}} = \frac{0.025x}{0.0038}\,. \tag{41}$$

On solving for x (to three decimal places), we get

$$x = 0.0760\,\text{mol} \cdot \text{L}^{-1}\,. \tag{42}$$

14 Problem 3: Grams-to-liters

This third problem is taken from the textbook *Chemistry: The Molecular Nature of Matter and Change* ([5], p. 120).

PROBLEM:

Given 0.10 grams of $Mg(OH)_2$ (a base) reacts completely with how many liters of 0.10 M HCl? The chemical equation for the reaction is

$$\mathrm{Mg(OH)_2(s) + 2HCl(aq) \longrightarrow MgCl(aq) + 2H_2O(l)}\,. \tag{43}$$

SOLUTION:

First, the diagram:

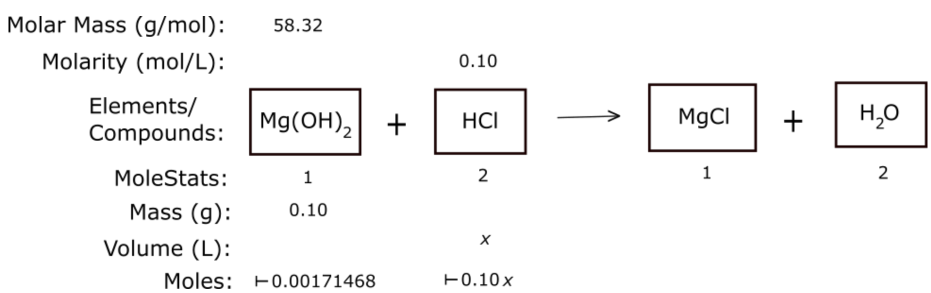

Figure 13. This graphic represents the neutralization of HCl acid with the base $Mg(OH)_2$. Note that x has unit of liters.

Next, we write down the mole proportion between columns 2 and 1:

$$\frac{2}{1} = \frac{\text{moles HCl}}{\text{moles Mg(OH)}_2} = \frac{0.10x}{0.00171468}\,. \tag{44}$$

On solving for x, we get

$$x = 3.4 \times 10^{-2}\,\mathrm{L}\,. \tag{45}$$

15 Problem 4: Iron Content in Ore Sample

This fourth problem is taken from *Chemical Principles: The Quest for Insight* ([1], p. F85).

PROBLEM: (paraphrase)

A sample of iron ore of mass 0.202 g is first dissolved in acid and then titrated with potassium permanganate in the folllowing reaction:

$$\mathrm{5Fe^{2+}(aq) + MnO_4{}^{-}(aq) + 8H^{+}(aq) \longrightarrow 5Fe^{3+}(aq) + Mn^{2+}(aq) + 4H_2O(l)} \tag{46}$$

If it takes 16.7 mL of 0.0108 M $KMnO_4$(aq) to reach the stoichiometric point (the point at which all the Fe^{2+} is consumed), what is the mass and percentage of iron in the sample?

SOLUTION:

We can calculate the percentage of iron in the sample after we have calculated the grams of iron that reacted with the permanganate. Now, we produce a diagram of the equation:

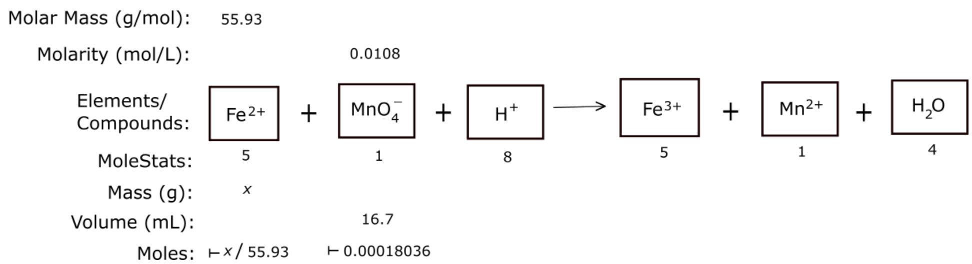

Figure 14. This graphic displays only enough numeric information to solve for x grams of Fe^{2+}.

Next, we write down our mole proportion between columns 1 and 2:

$$\frac{5}{1} = \frac{\text{moles Fe}^{2+}}{\text{moles MnO}_4^-} = \frac{x/55.93}{0.00018036}\,. \tag{47}$$

On solving for x, we get

$$x = 0.0504\,\text{g}\,. \tag{48}$$

Therefore, the percentage of iron in the ore sample is

$$\%\ \text{Fe} = \frac{0.0504\,\text{g}}{0.202\ \text{g}} \times 100\% = 25.0\%\,. \tag{49}$$

16 Problem 5: Limiting Reactant 1

This next problem is taken from the YouTube chemistry course presented by Tyler DeWitt at

https://www.youtube.com/watch?v=nZOVR8EMwRU

Up to this point, we have seen how two reactants will be totally consumed if they are combined according to the molar ratio given by the corresponding Molestats numbers. What would happen, then, if these reactants are combined in some other molar ratio? In this case, one of the reactants will be left, while the other consumed, and the remaining reactant is called the *excess reactant* and the consumed reactant the *limiting reactant.*

Tyler gave this simple example by use of a baking recipe. Suppose that a certain recipe for bread rolls requires 3 cups of flour and one cup of water to produce 5 bread rolls. What is the limiting ingredient if one has 6 cups of flour and 3 cups of water available? How many bread rolls can be made with the given ingredients?

Either both ingredients are consumed, or one of the ingredients is consumed (the limiting ingredient) while the other is not consumed (the excess ingredient). Now we draw the diagram.

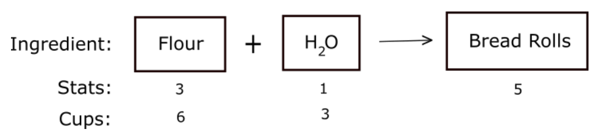

Figure 15a. This graphic displays all the given information.

The next step is to replace cup quantities in one column (say the flour column) by a variable, say x, for the purpose of determining exactly how much of that corresponding ingredient **would** be needed to be fully consumed in the reaction.[19] See the figure below.

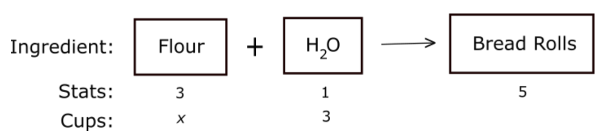

Figure 15b. This graphic displays the same information as the last figure, except that the 6 cups of flour has been replaced by the variable x, to be solved for, when both reactants (ingredients) are totally consumed.

Our relevant fundamental proportion for this problem is

$$\frac{3}{1} = \frac{\text{cups flour}}{\text{cups water}} = \frac{x}{3}\,. \tag{50}$$

Solving for x, we get $x = 9$ cups. So, to use up all the water, we'd need 9 cups of flour, though we have only 6 cups. Therefore, the flour is the limiting reactant (ingredient) in this baking project. So, by looking at Figure 15a, we can see that if we use all 6 cups of flour and (reasoning proportionally) using 2 cups of water, we can produce 10 bread rolls.

After finishing this example problem, Tyler presented a real chemistry problem, which I now state in paraphrase: In the process of making ammonia (NH_3)

[19]For an example of someone else using this method, see http://www2.ucdsb.on.ca/tiss/stretton/CHEM1/stoich7.html.

from the reactants 3.2 moles of N_2 and 5.4 moles of hydrogen (H_2), what is the excess reactant and how many moles of it are left over? We've seen this before in (1). Next, make a diagram with one mole count replaced by a x.

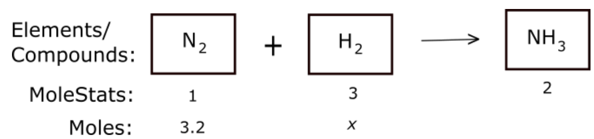

Figure 16. This graphic represents the complete reaction of 3.2 moles of N_2 with x moles of H_2.

Now, we write down the mole proportion between columns 2 and 1:

$$\frac{3}{1} = \frac{\text{moles } H_2}{\text{moles } N_2} = \frac{x}{3.2}\,. \tag{51}$$

On solving for x, we get

$$x = 9.6\,\text{mol}\,. \tag{52}$$

So, to use all the nitrogen, we need 9.6 moles of hydrogen, which is 4.2 moles more than we have on hand. Therefore, the hydrogen is the limiting reactant. Now, we can't use all of the nitrogen (thus it's the excess reactant), but we can use all the hydrogen, and if we do, how much nitrogen is needed (in moles)? We can determine this be solving the simple proportion $1 : 3 :: x : 5.4$, or, as an equation

$$\frac{1}{3} = \frac{\text{moles } N_2}{\text{moles } H_2} = \frac{x}{5.4}\,, \tag{53}$$

which has solution: $x = 1.8$ moles.

Using the 5.4 moles of hydrogen, how much nitrogen is left over and how much ammonia can be made? The answer to the first question is easy: The excess = 3.2 mol - 1.8 mol = 1.4 mol. (See Figure 17.)

Next, we write down our mole proportion between columns 1 and 2:

$$\frac{2}{3} = \frac{\text{moles } NH_3}{\text{moles } H_2} = \frac{x}{5.4}\,, \tag{54}$$

which has solution $x = 3.6$ mol.

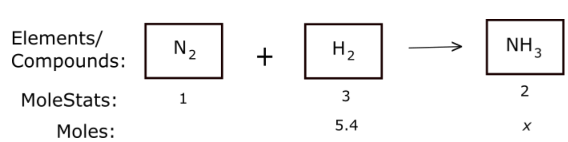

Figure 17. This graphic represents the complete reaction of 5.4 moles of H_2 to produce x moles of NH_3.

To answer the second part of the question, again, we make a diagram, but this time with a variable in the mole count for ammonia.

17 Problem 6: Limiting Reactant 2

This next problem is taken from *Chemistry with Melissa Maribel.*[20] She also covers the production of ammonia but begins with known masses reactants. I state her version of the problem in paraphrase:

If 14.32 g of N_2 reacts with 4.21 g of H_2 to produce NH_3, what is the limiting reactant?

First, we repeat the balanced equation of the reaction Eq. (1). Next, we make a diagram with one of the mole counts replaced by a variable.

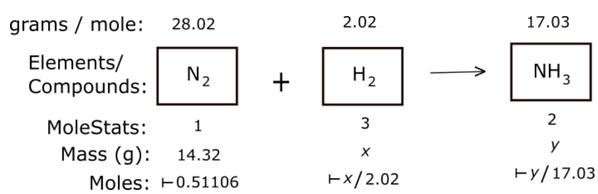

Figure 18. This graphic represents the complete reaction of 14.32 grams of N_2 with x grams of H_2 to produce y grams of NH_3.

Next, we write down our mole proportion between columns 1 and 2:

$$\frac{3}{1} = \frac{\text{moles } H_2}{\text{moles } N_2} = \frac{x/2.02}{0.5078}\,. \tag{55}$$

For which we get $x = 3.10$ g. So, to use all the nitrogen, we have an excess of hydrogen, which makes the nitrogen the limiting reactant.

To calculate how much ammnonia would be produced, we setup another mole proportion, this time between columns 3 and 1:

$$\frac{2}{1} = \frac{\text{moles } NH_3}{\text{moles } N_2} = \frac{y/17.03}{0.51106}\,, \tag{56}$$

which has solution: $y = 17.41$ grams.

One last point to make: The YouTube videos revealed to me two different, though equivalent, definitions of *limiting reactant.*

Definition 1: The *limiting reactant* is that reactant that is consumed, leaving a portion of the other reactant unconsumed.

Definition 2: The *limiting reactant* is that reactant (of only two reactants) which, when fully consumed, produces the lesser amount of product.

Generally, I prefer to use the first definition, but both are useful.

[20]Found at https://www.youtube.com/watch?v=ymCZ2ShhBAw.

18 Comments on Stoichiometric Strategies

I guess it's because my major training is in mathematics, rather than chemistry, that I find the usual chemists's approach to stoichiometric strategies (the magic of multiple products of conversion factors) so foreign to my temperament.

I am reminded of the scene from *The Magnificient Seven* in which Chris (played by Yul Brynner), the leader of the seven defenders of the small Mexican village, along with Vin (played by Steve McQueen), confronted Calvera (played by Eli Wallach), the leader of the bandits attacking the village, about this very controversy:

Calvera (after doing a bit of arithmetic): Hm, seven. Somehow I don't think you've solved my stoichiometry problem.
Chris: Solving your problems *your way* isn't our line.
Vin: We deal in algebra, friend. Actual equations!
Calvera: So do I. We're in the same business: multiple conversion factors, eh?
Vin: Only as competitors.

Humor aside, my complaint is quite general: I prefer to use algebraic equations to solve stoichiometry problems, rather than to search about the ether, hoping to find the right sequence of conversion factors to multiply out.[21] In a later paper of this series, we will see problems so complicated that use of a system of algebraic equations for solving stoiciometry problems can hardly be avoided.

19 Problem 7: Volume-to-Moles Problem

This problem is taken from *Chemistry–Unit 5: Stoichiometry: Practice Problems*, found online at

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https://www.dsd.k12.wi.us/faculty/SBAXTER/Unit%205
                              %20Practice%20Problems%20(answers).pdf
```

PROBLEM:

6) How many liters of carbon monoxide at STP are needed to react with 4.80 g of O_2 to produce CO_2? The equation for the reaction is

$$2CO(g) + O_2(g) \rightarrow 2CO_2(g)\,. \tag{57}$$

SOLUTION:

To solve this problem, we need to know that every ideal gas at STP contains about 22.4 liters per mole of gas molecules. Of course, we will model our carbon monoxide gas as ideal for this problem. As usual, we present a graphic for assistance.

[21] Without a mole proportion equation to clarify things, how do I know if the stoichiometric ratio, being used as one of many 'conversion factors', is a/b or b/a?

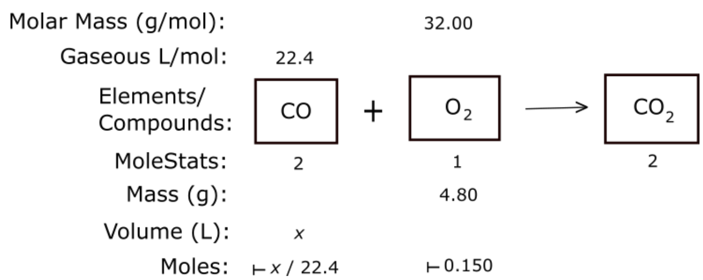

Figure 19. A sparce diagram. The CO is contained at STP.

Next, we write down our mole proportion from columns 1 and 2:

$$\frac{2}{1} = \frac{\text{moles CO}}{\text{moles O}_2} = \frac{x/22.4}{0.150}\,. \tag{58}$$

On solving for x, we get

$$x = 6.72\,\text{L}\,. \tag{59}$$

Comment:

In computer science there is the concept of the *sparce array*, which is an array in which most of the entries are zero. In *Scheme* there is the concept of the *sparce diagram*, which is a diagram in which many entries are left blank, almost always because the information the blank entries carry is irrelevant to solving the problem. One exception to this rule is that I *usually* include all the MoleStats numbers, even when they're not all used.

20 Problem 8: Volume-to-Volume Problem

This problem is taken from *Solving Stoichiometry Problems* found online at

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http://www.csun.edu/~psk17793/G%20Chemistry
                /solving_stoichiometry_problems.htm
```

PROBLEM:

4) How many liters of SO_2 will be produced from 26.9 L O_2? The equation for the reaction is

$$S_2(g) + 2O_2(g) \rightarrow 2SO_2(g). \tag{60}$$

SOLUTION:

To solve this problem, we need to make some simplifying assumptions. First, we'll model both gases as ideal. Second, we'll assume that both gases are at the same temperature and pressure. Now, all ideal gases have the same volume per mole, which in this case we'll designate as v.[22]

As usual, we present a graphic for assistance.

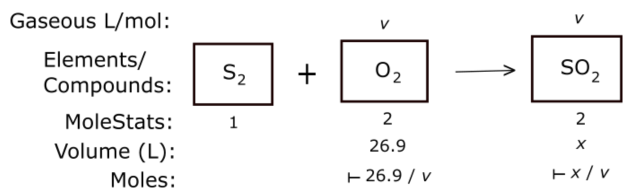

Figure 20. The SO_2 is collected at STP.

Next, we write down the mole proportion between columns 3 and 2:

$$\frac{2}{2} = \frac{\text{moles } SO_2}{\text{moles } O_2} = \frac{x/v}{26.9/v}\,. \tag{61}$$

On solving for x, we get

$$x = 26.9\,\mathrm{L}\,. \tag{62}$$

21 Problem 9: Solving for Mystery Element X

This problem is taken from *Chemical Principles: The Quest for Insight* ([1], Problem L21, p. F87).

PROBLEM:

The compound $XCl_2(NH_3)_2$ can be formed by reacting XCl_4 with NH_3. Suppose that 3.571 g of XCl_4 reacts with excess NH_3 to give 3.180 g of $XCl_2(NH_3)_2$. What is element X?

SOLUTION:

We begin with a balanced equation for the reaction. I include a presumptive chlorine term on the product side to make this feasible.

$$XCl_4 + 2NH_3 \longrightarrow XCl_2(NH_3)_2 + Cl_2\,. \tag{63}$$

One thing in our favor toward solving this problem is that, by inspection of the elements in the periodic table, the atomic masses are unique. Thus, if we can find the atomic mass of element X (in g·mol^{-1}), we should be able to quickly

[22] The problem was solved on the website by assuming the gases are at STP, but I won't make that assumption.

identify the element in the table. Therefore, we let x represent the atomic mass of element X. If we can solve for x, we should be able to identify element X.

Now, since the molecular mass of Cl is 35.45 g·mol^{-1}, and that of NH_3 is 17.03 g·mol^{-1}, then the molecular mass of XCl_4 is $x + 4(35.45) = x + 141.8$, and for $XCl_2(NH_3)_2$ we get a molecular mass of $x + 2(35.45) + 2(17.03) = x + 104.96$.

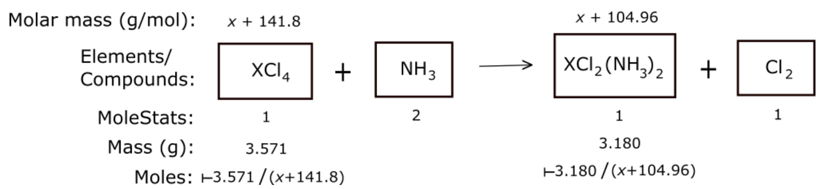

Figure 21. This graphic displays a logically ‘extra’ Cl_2 term.

Next, we write down our mole proportion between columns 1 and 3:

$$\frac{1}{1} = \frac{\text{moles } XCl_4}{\text{moles } XCl_2(NH_3)_2} = \frac{3.571/(x + 141.8)}{3.181/(x + 104.96)} \,. \tag{64}$$

Wolframalpha gives the solution for x as

$$x = 195.52\,\mathrm{g}\,. \tag{65}$$

The element in the periodic table whose atomic mass is closet to x is Platinum.

22 Problem 10: Analyzing Vitamin C

This problem is taken from *Chemical Principles: The Quest for Insight* ([1], Problem L16, p. F87).

PROBLEM:

A tablet of vitamin C was analyzed to determine whether it did in fact contain, as the manufacturer claimed, 1.0 g of the vitamin. One tablet was dissolved in water to form 100.00 mL of solution, and 10.0 mL of solution was titrated with iodine (as potassium triiodide). It required 10.1 mL of 0.0521 M I_3^-(aq) to reach the stoichoimetric point[23] in the titration. Given that 1 mol I_3^- reacts with 1 mol vitamin C in the reaction, is the manufacturer's claim correct?

SOLUTION:

We begin with recognizing what we must actually show. We must show 1) that our calculation of the quantity of vitamin C in the original solution must be 10 times that in the titrated solution, and 2) that our calculation for the vitamin

[23]That is, when all the vitamin C was consumed.

C contents of the titrated solution must lie between 1.04 g and 0.95 g in order to round to 1.0 g.

Normally, at this point I'd produce a balanced chemical equation of the reaction, but this time I won't, principally because the products of the reaction aren't given because they're not needed. What we are given instead is the stoichiometric ratio of vitamin C consumption to I_3^- consumption being 1 : 1. But we can, and should, still produce a diagram of the reaction.

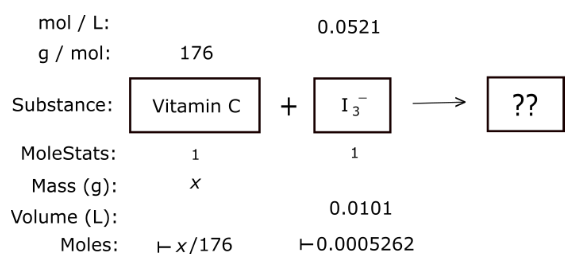

Figure 22. Note: The volume has been converted from mL to L.

But first a word about the notation in the diagram: When I use the double question mark '??', I refer to a quantity that I'm **not** interesting in knowing, probably because it's irrelevant to the problem.

Next, we write down the mole proportion between columns 1 and 2:

$$\frac{1}{1} = \frac{\text{moles Vit C}}{\text{moles } I_3{}^-} = \frac{x/176}{0.0005262} \,. \tag{66}$$

Solving for x, we get

$$x_{10} = 0.0926 \, \text{g} \,, \tag{67}$$

where x_{10} is the mass corresponding to the 10.0 mL volume. Therefore, we multiply it by ten to get the 100.00 mL mass (approximately):

$$x_{100} = 0.926 \, \text{g} \,. \tag{68}$$

However, this values lies outside the predetermined appropriate range. Therefore, the answer to the question posed is No.

23 Problem 11: Molecules-to-Molecules

This problem is taken from the online pdf file

`https://www.dsd.k12.wi.us/faculty/SBAXTER/Unit %205%20Practice%20Problems%20(answers).pdf`

PROBLEM 9:

Given the balanced equation $2H_2 + O_2 \rightarrow 2H_2O$, how many molecules of water are produced from 2.0×10^{23} molecules of oxygen?

SOLUTION:

We begin with a diagram.

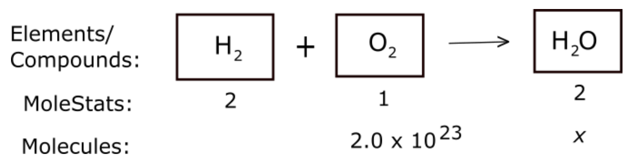

Figure 23. This graphic displays the products in the form of a lumped term, which is of no particular interest to us in this problem.

Next, we write down our mole proportion between columns 2 and 3:

$$\frac{2}{1} = \frac{x}{2.0 \times 10^{23} \text{ molecules}} . \tag{69}$$

Solving for x, we get

$$x = 4.0 \times 10^{23} \text{ molecules} . \tag{70}$$

24 Problem 12: Moles-to-Grams

This problem is taken from the online pdf file

`http://www.msduncanchem.com/Unit_9/unit_9_ws_reg.pdf`

PROBLEM 5, p, 3:

Titanium is a transition metal used in many alloys because it is extremely strong and lightweight. Titanium tetrachloride ($TiCl_4$) is extracted from titanium oxide using chlorine and carbon.

$$\_\_TiO_2 + \_\_C + \_\_Cl_2 \longrightarrow \_\_TiCl_4 + \_\_CO_2 . \tag{71}$$

If you begin with 1.25 moles of TiO_2, what mass of Cl_2 gas is needed? (Ans: 178 g Cl_2.)

SOLUTION:

We begin by balancing Eq. (71).

$$TiO_2 + C + 2Cl_2 \longrightarrow TiCl_4 + CO_2 . \tag{72}$$

Once again, a diagram.

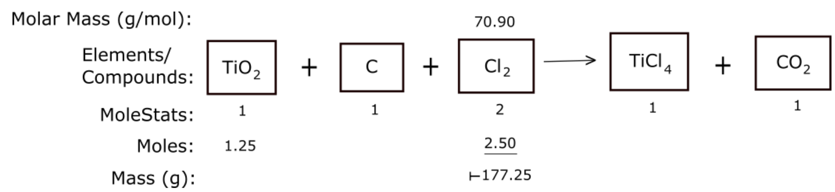

Figure 24. Another sparce graphic that only displays relevant data. Remember that the rule for the underline markup is to imply that at least one piece of data was used from a different column to derive the underlined number.

The work is already finished. The value for x the grams of chlorine is 177 g to three decimal places.

25 Appendix A: Relative Molecular Masses

Atomic masses are given in terms of grams per mole (g·mol^{-1}). For the compounds, I used the values given by
https://www.convertunits.com/

Ag — 107.87 (Silver)
AgBr — 187.77 (silver bromide)
AgCl — 143.32 (silver chloride)
Ag_2CrO_4 — 331.73 (silver chromate)
$AgNO_3$ — 169.87 (silver nitrate)

Al — 26.98 (Aluminum)
Al_2O_3 — 101.96
$Al(OH)_3$ — 78.00 (aluminum hydroxide)
AlC_3 — 133.34
$Al_2(CrO_4)_3$ — 401.94
$Al_2(SO_4)_3$ — 342.15

As — 74.92 (Arsenic)
As_4O_6 — 395.68

B — 10.81 (Boron)
B_2H_6 — 27.67
B_2O_3 — 69.62

Ba — 137.33 (Barium)
$BaCl_2$ — 208.23 (barium chloride)
$Ba(OH)_2$ — 171.34
$Ba(NO_3)_2$ — 545.80
$BaSO_4$ — 233.39 (barium sulfate)

Be — 9.01 (Beryllium)

Br — 79.90 (Bromine)
Br_2 — 159.81

C — 12.01 (Carbon)
CCl_4 — 153.82 (Carbon tetraflouride)
$CHCl_3$ — 119.38 (Chloroform)
CBr_2Cl_2 — 242.72
CH_3OH — 32.04

CH_3COOH — 60.05
CO — 28.01
CO_2 — 44.01
COC_2 — 98.92 (phosgene)
CH_2O — 30.03
CH_5NO_2 — 63.01 (ammonia formate)
C_2H_2 — 26.04 (acetylene)
C_2H_6 — 30.07 (ethane)
C_2H_4O — 44.53 (...)
C_3H_6O — 50.08
$C_3H_6O_3$ — 90.08 (lactic acid)
$C_3H_8O_3$ — 92.09
$C_6H_{12}O_6$ — 180.16
$C_6H_5CO_2K$ — 160.21 (potassium benzoate)
$C_3H_5(ONO_2)_3$ — 227.09 (nitroglycerin)
$C_7H_5(NO_2)_3$ — 227.13
CH_3 — 15.03 (methyl radical)
CH_4 — 16.04 (methane)
CH_3OH — 32.04
C_3H_8 — 44.10 (propane)
C_4H_8 — 56.11 (butene)
C_4H_{10} — 58.12 (butane)
C_5H_{10} — 70.13 (?)
C_5H_{12} — 72.15 (pentane)
C_8H_{18} — 114.23 (octane)

Ca — 40.08
$CaBr_2$ — 199.89
CaC_2 — 64.10
$CaCl_2$ — 110.98
$CaCl_2 \cdot 2\,(H_2O)$ — 128.99
CaO — 56.08 (calcium oxide)
$Ca(OH)_2$ — 74.09
$Ca_2(PO_3)_2$ — 270.10
$Ca_3(PO_3)_2$ — 310.18 (calcium phosphate)
$CaCO_3$ — 100.09
$CaSO_4$ — 136.14
$CaSiO_3$ — 116.16 (calcium metasilicate)

Cl — 35.45 (Chlorine)
Cl_2 — 70.91

Co — 58.93 (Cobalt)
$CoCl_2$ — 129.84 (cobalt chloride)

Cr — 52.00 (Chromium)
Cr_2O_3 — 152.00
$Cr(NO_3)_2$ — 176.01

Cs — 132.91 (Cesium)

Cu — 65.39
$CuCl_2$ — 134.45
$Cu(OH)_2$ — 97.56
$Cu(NO_3)_2$ — 183.56 (copper(II) nitrate)
Cu_2S — 159.16 (copper(I) sulfide)
Cu_2O — 143.09 (copper(I) oxide)
$CuSO_4$ — 159.61

F — 19.00
F_2 — 38.00

Fe — 55.93 (Iron)
$FeCl_2$ — 126.75
$FeCl_3$ — 162.20
Fe_2O_3 — 159.69 (iron(III) oxide)
$FeSO_4$ – 151.91
$Fe_2(SO_4)_3$ – 399.88
FeS – 87.91 (iron(II) sulfide)
$FeTiO_3$ – 151.71 (iron(II) titanate)

Ga — 69.73
Ga_2O_3 — 187.44 (gallium(III) oxide)

H — 1.01
H_2 — 2.02
HBO_2 — 43.82
HBr — 80.91 (hydrobromic acid)
$H_2C_2O_4$ — 90.03
$H_2C_4H_4O_6$ — 150.087
HCN — 27.06
H_3BO_2 — 45.83
HCl — 36.46
$HClO_4$ — 100.56 (perchloric acid)
HF — 20.01

HI — 127.91 (hydrogen iodide)
H_2O — 18.01
H_2O_2 — 34.01
HNO_3 — 63.01
H_3PO_4 — 24.31
H_2S — 34.08
H_2SO_4 — 98.08
H_2SO_3 — 82.01

Hf — 178.49 (Hafnium)

Hg — 200.59 (Mercury)
Hg_2Br_2 — 560.99 (mercurous bromide)
Hg_2Cl_2 — 472.09 (mercurous chloride)

I — 126.90 (Iodine)
I_2O_5 — 333.81 (diiodine pentoxide)

K — 39.10
KCl — 74.55
K_2CrO_4 — 194.19
$K_2Cr_2O_7$ — 294.18
KCN — 65.21
$K_4Fe(CN)_6$ —v368.34
K_2HPO_4 — 174.18
KIO_3 — 214.00 (potassium iodate)
K_3PO_4 — 212.27
KO_2 — 71.10
KOH — 56.10
$KMnO_4$ — 158.03
KNO_2 — 85.10 (potassium nitrite)
KNO_3 — 101.10 (potassium nitrate)
K_2SO_3 — 158.26 (potassium sulfite)
K_2SO_4 — 174.26 (potassium sulfate)

Li — 6.94
$LiBr$ — 86.85 (lithium bromide)
$LiCl$ — 42.39 (lithium chloride)
$LiClO_4$ — 106.39 (lithium perchlorate)
Li_2CO_3 — 73.89
L_3N — 34.83
$LiNO_3$ — 68.95

$LiOH$ — 23.95 (lithium hydroxide)
Lil_2SO_4 — 109.94

Mg — 24.31
$MgCl$ — 59.76
$MgCl_2$ — 95.21
MgF — 43.30 (magnesium flouride)
$MgCO_3$ — 83.31 (magnesium carbonate)
Mg_3N_2 — 100.93 (magnesium nitride)
MgO — 40.30 (magnesium oxide)
$Mg(OH)_2$ — 58.32
$MgSO_4$ — 120.37

Mn — 54.94 (manganese)
MnO_2 — 86.94
$Mn(NO_3)_3$ —240.95 (manganese (III) nitrate)
Mn_2S_3 —206.07 (manganese (III) sulfide)

N — 14.01
N_2 — 28.01
N_2l_2 — 30.03
NH_3 — 17.03
NH_4 — 18.01
$(NH_4)_2Cr_2O_7$ — 252.06
$(NH_4)_2CO_3$ — 96.09 (ammonium carbonate)
$(NH_4)Cl$ — 53.49
$(NH_4)ClO_4$ — 117.49
NH_4OH — 35.05
NH_4NO_3 — 80.04
NO — 30.01
NO_2 — 46.01
N_2O_5 — 108.01

Na — 23.00
$NaBr$ — 102.89 (sodium bromide)
$NaCl$ — 58.44 (sodium chloride)
$NaClO_4$ — 58.44 (sodium perchlorate)
$NaCN$ — 49.01 (sodium cyanide)
Na_2CO_3 —105.99 (sodium carbonate)
$Na_2C_2O_4$ — 134.00
Na_2CrO_4 — 161.97
$Na_3C_6H_5O_7$ — 258.07
NaF — 41.99 (sodium flouride)

Na_3PO_4 — 163.94
$NaHCO_3$ — 84.01
$NaIO_3$ — 197.89 (sodium iodate)
NaN_3 — 65.01
$NaNO_3$ — 84.99
$NaKC_4H_4O_6$ — 210.16
NaOH — 40.00
Na_2SO_4 — 142.04
$Na_2S_2O_3$ — 158.11

Ne —20.18

O —16.00
O_2 —32.00
O_3 —48.00

P — 30.97
P_4 — 123.90
P_4H_{10} — 133.97 (phosphorus pentoxide)
PH_3 — 34.00 (phosphine)
PH_4I — 161.91
P_2I_4 — 569.57

Pb —207.20
$PbCl_2$ — 278.11 (lead(II) chloride)
$PbCrO_4$ — 323.19
PbS — 239.27
PbO — 223.20
$Pb(SO_4)_2$ — 399.33
$Pb(NO_3)_2$ — 331.21 (lead(II) nitrate)
$Pb(NO_3)_4$ — 455.22

Ra —226.03 (Radium)

Rb — 84.87

S — 32.07
SO_2 — 64.06
$SO_4{}^{2-}$ — 96.06

Sb — 121.76 (Antinomy)

Sb_2O_3 — 291.52

Sc — 44.96 (Scandium)

Si — 28.09
SiO_2 — 60.08

Sr — 87.62 (strontium)
SrO — 103.62 (strontium oxide)

Ti — 47.88
$TiCl_4$ — 198.68 (titanium (IV) chloride)
Ti_2O_2— 127.73

U — 238.03
UF_6 — 352.02
U_3O_8 — 842.08

Y — 88.91 (Yttrium)

Zr —91.22

Zn — 65.39
$ZnCl_2$ — 136.29
$Zn(NO_3)_2$ — 189.39

26 Appendix B: Algebra used to Balance Chemical Equations

Now we're going to use the methods of algebra to solve for the coefficients to unbalanced chemical equations. For some reactions this is not possible as there are more than one possible combination of coefficients that will solve the equations. But for the given reaction below, the coefficients are unique up to an overall multiplicative factor.

Problem: Using algebraic methods, balance the following unbalance chemical equation:

$$\mathrm{KCN} + \mathrm{FeCl_2} \rightarrow \mathrm{K_4Fe(CN)_6} + \mathrm{KCl}\,. \tag{73}$$

Goal Statement: Find the coefficients that balance all the elements on the equation.

Now, we could provide each term its own coefficient to solve for. such as, So, (73) becomes

$$x_1\mathrm{KCN} + x_2\mathrm{FeCl_2} = x_3\mathrm{K_4Fe(CN)_6} + x_4\mathrm{KCl}\,. \tag{74}$$

However, we only need to solve for three unknowns since the coefficients are only determined up to their mutual ratios.

Therefore, we can simplify the problem by setting any one of the coefficients to unity, Say we choose x_1 for that. Thus (74) becomes

$$x_1\mathrm{KCN} + x_2\mathrm{FeCl_2} = x_3\mathrm{K_4Fe(CN)_6} + \mathrm{KCl}\,. \tag{75}$$

Now, we can think of this as a conservational problem: The total number of each element is conserved going from the left-hand side to the right-hand side.[24] Every total is equal to the sum of its parts. What are the parts, then? The parts are the contributions of the particular element from each term. Therefore we can write the conservational equation for Potassium, K, yielding

$$\text{Total K on LHS} = \text{Total K on RHS} \tag{76}$$

and the two others we need follow similarly. Then summing up and equating the term-wise contributions for K, N, and Fe gives

$$\begin{array}{lcccccccc} \mathrm{K} & : & 1x_1 & + & 0x_2 & = & 4x_3 & + & 1 \\ \mathrm{N} & : & 1x_1 & + & 0x_2 & = & 6x_3 & + & 0 \\ \mathrm{Fe} & : & 0x_1 & + & 1x_2 & = & 1x_3 & + & 0 \end{array} \tag{77}$$

There are a number of ways to solve (77). One way is to subtract the first equation from the second, yielding $x_3 = \frac{1}{2}$. Substituting this value into the second gives $x_1 = 3$. And from the third equation we get $x_2 = x_3 = \frac{1}{2}$. Thus (75) becomes

$$3\mathrm{KCN} + \tfrac{1}{2}\mathrm{FeCl_2} = \tfrac{1}{2}\mathrm{K_4Fe(CN)_6} + \mathrm{KCl}\,. \tag{78}$$

[24]In saying 'number', we can think in terms of individual atoms or in terms of moles.

On multiplying this through by 2 we get

$$6\text{KCN} + \text{FeCl}_2 = \text{K}_4\text{Fe(CN)}_6 + 2\text{KCl}\,. \tag{79}$$

We got the coefficients for K, N, and Fe, sure enough, but we're not finished yet. According to the goal statement, we still need to verify that the coefficients work for C and Cl, which they do, and this can easily be proved by inspection.

27 Appendix C: A Short History of *Scheme*

What brings me to adapt *Scheme* to stoichiometry is my desire to tutor chemistry in the near future. What brought me to develop *Scheme* originally was my frustration with the problem-solving methods I learned for algebra word problems in high school, so very long ago.

As I look back on it now, I think that my basic approach to solving word problems in high school, and for years beyond, was to
1) Look for variables (unknowns).
2) Look for relationships (usually equations) on those variables.
3) Solve the system.
I'll refer to this set of heuristics as the *naive approach*.

Now, these heuristics are not actually wrong, but my experience over the last four decades has been that they aren't exactly efficient, either. It took me a long time to realize that my 'logically correct' heuristics just didn't seem to work — at least not for me. It remained for me a frustrating logical paradox.

But slowly over the years after I graduated college, I began to see a different way to approach these problems. Although I don't remember the details on how I came to resolve the impasse on algebra problem solving, I do remember that it all boiled down to answering just two logical questions:

1) How should one best define an algebra word problem? and
2) How should one build efficient heuristics upon that definition?

To the first question, I answer: An algebra word problem is the translation of an algebra problem (given in some natural language) into one or more equations (and/or inequalities), in one or more unknowns, and then solve for those unknowns. (For the purpose of introducing *Scheme*, I will ignore the complications of word problems with inequalities, because they don't add anything to the overall concepts I need to present.)

My single greatest insight was that virtually all equations in algebra word problems can be categorized into a handful of easily recognizable types. The following are the most common types of **equation generators** I have found:
1) Every total is **equal** to the sum of its parts.
2) Every invariant Q in a 'before-and-after' process generates the **equation** $Q_i = Q_f$, where i is the initial state and f is the final state.
3) A proportion: Every proportion claims the **equality** of two fractions (ratios).
4) Formulas (**equations**) from science or geometry, such $V = IR$ from physics, or area of a circle $= \pi r^2$ from geometry.
5) **Equations** that constrain the unknowns of the problem, but are not of the previous four types. I refer to such **equations** as *constituitive relations*, such as Sally's age = one more than twice John's age.

The next greatest insight I had in my search for an efficient plan to solve word problem was to **not** rush to find unknowns, but rather, after finding a word equation to translate, execute on it a *step-wise refinement* of the word equation until in its final form it exists as a pure algebraic statement. This procedure has a name: it's called the *top-down-approach with step-wise refinement*[25] to problem solving (for those who remember the movie *Contact* — 'small moves').

For example, consider the word problem (often referred to as a 'rate' problem):

► Printer #1 can print a 100 copies of a document in 3.4 hours and Printer #2 can print out the same print job in 2.5 hours. How long will it take for the print job to complete if both printers work on the job together, starting and stopping at the same time?

Now, we resist the urge to rush in to find unknowns. Instead, first, we look for totals and parts. Are there any? (Look hard to find them!) Yes, there are — or, rather, there is. There is a total of one job being done by two contributing printers: Printer 1 and Printer 2. All right, we know that every total is **equal** to the sum of its parts. So, let's introduce the shorthand 'part of job done by' → PJDB. Then our highest-level equation is

$$1 \text{ job} = (\text{PJDB Printer 1}) + (\text{PJDB Printer 2})\,. \tag{80}$$

Generally speaking, the amount of production of a machine over time is given by the product of the rate R at which it produces output × the time T it runs. Therefore, let R_1 be the average rate at which Printer 1 can work, which is 1 job/3.4 hours.[26] Likewise, R_2 is the average rate at which Printer 2 can work, which is 1 job/2.5 hours.[27] Now, the most general expression we can write for the refinement of the last equation is (suppressing units)

$$1 = R_1T_1 + R_2T_2\,, \tag{81}$$

where T_1 and T_2 are the respective times that Printer 1 and Printer 2 are operating, which, in this particular problem, are the same number, we'll just call T. So, the last equation becomes

$$1 = (R_1 + R_2)T\,. \tag{82}$$

So, now we can solve for time T, the time to finish the job:

$$T = \frac{1}{R_1 + R_2}\,. \tag{83}$$

Using the numbers given,

$$T = \frac{1}{(1/3.4) + (1/2.5)} = 1.44\,[\text{hours}]\,. \tag{84}$$

[25] This is a technique I learned for developing algorithms in a computer programming class.

[26] How to decide the units? Should it be job/hour or hour/job? It must be the former because we must have *job* in the numerator to match *job* being in the numerator on the left-hand side of the equation.

[27] We are employing the Zeroth Rule of Problem Solving to make the simplifying assumption that the average rate will be accurate for arbitrarily long or short time intervals.

Thus, solving the problem in 'small moves' makes it easier to solve. And, by the way, if one has to solve a similar problem in which the constitutitive relation on T_1 and T_2 is more complicated than just $T_1 = T_2$, then that can be easily dealt with, too.

Consider the following problem, in which there is a non-trivial constituitive relation on the time variables of two people doing a cooperative job (often referred to as a 'work' problem):

► Steve can mow a lawn in three hours and Joe can mow the same lawn in two hours. How long will each of them take to mow the lawn if they both work on it together, except that Joe works 20 minutes before Steve starts to work?

Clearly, we won't be able to just mindlessly apply the so-called 'work formula' in (83) in this case, since $T_1 \neq T_2$. If we let T_1 be Joe's time and T_2 be Steve's time, then we can write down the constituitive relation

$$T_1 = T_2 + 1/3\,, \tag{85}$$

where time is measured in hours. However, this same relationship can be obtained by taking a 'total is the sum of its parts' analysis. (How? Hint: Use the top-down approach; partition the timeline.)

I didn't employ a diagram for the previous problem, but I will for this one.

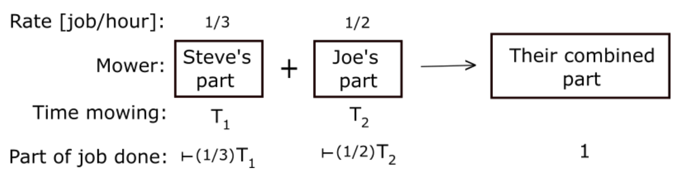

Figure C1. Diagramming this work problem as a total is the sum of its parts. The turnstile before a value or expression means that it was computed using only information in the same column.

Observation on the above figure: Viewed from the perspective of object-oriented programming, the boxes represent *objects* that have rates and times as data, or *properties*, and (from the bottom line) has the function (rate × time) as a *method*. The columns represent an *encapsulation* of an objects's data and the functions on that data. So, instead of placing relevant information about a given problem scattered about on the page, *Scheme*'s diagramatic encapsulation keeps an object's related data together in one column.

All this fussiness about encapsulation is about presenting information to yourself and to someone else so that the information is as easy to comprehend as is possible. English composition has similar rules to accomplish the same goal. As Struck and White prescribed one of those rules: "Keep related words together" (*The Elements of Style*).

Back in the mid-1980s, when I was first formulating the rules of *Scheme*,[28] I used a procedure to solve a problem similar to those just presented, and was able for the first time, not only to solve the problem, but also to feel as though I truly understood the solution. And *then* I knew that I had the beginnings of a scheme to solve algebra word problems with confidence. Since then, I have scoured textbooks and the Internet to find challenging/difficult word problems on which to test *Scheme*. And I admit, some of them *were* challenging to me. To date, I've written over 30 short unpublished papers using *Scheme* to solve algebra word problems, this series of stoichiometry papers not included.

I have a few general comments to make about *Scheme* at this point.

► Part of the success of *Scheme* comes from its combination of eclectic heuristics for solving word problems (already discussed) and its own terminology and diagramming protocols. Let's deal with the terminology first. If you're already familiar with current nomenclature on algebra word problems, no doubt you know of 'rate' problems, 'work' problems, and 'mixture' problems.

As an example of a mixture problem, consider the coin problem given on page 4, in which we consider a pile of coins as a mixture of pile of nickels and a pile of dimes. The rates involved are the conversion factors that convert the unit coin of a given denomination to its dollar value.

Now, I'm not against having these terms ('rate', 'work', 'mixture'), but I did want *Scheme* to have a single term rule them all (and that has a nice ring to it). To that end, I invented (or, probably only reinvented) the term *Mixed-Rate* problem.

Definition: A *mixed-rate problem* is an algebra problem in which two or more 'machines' work together, at generally different rates, to accomplish a common goal.

The point is that this definition is designed to cover, all at once, rate problems, work problems, and mixture problems. But there's a cost to co-opting the word 'machine' for such abstraction, which is that often the things to which it applies will not look very much like a 'machine'.

Definition: A *simple machine* is a named entity in an algebra problem that converts one unit into another unit by a fixed rate, R, say, and R is also said to be a *conversion factor*. To every conversion factor R is associated some quantity Q with unit the same as the denominator unit of R.

Now a generalization:

Definition: A *machine* is a named entity in an algebra problem that has associated to it one or more conversion factors, R_i, each having its own associated quantity Q_i.

For example, Alice's nutritionist has recommended to her how many calories

[28] The name *Scheme* is only a recent addition to the set of rules (heuristics), being adopted around 2015.

and grams of protein she should consume every morning. One morning she wants to fulfill these requirement with milk and a bagel. She has to calculate how much of each to consume. Her nutritionist has provided for her a table that contains for each of these food items the calories per ounce and the grams of protein per ounce. Thus, the 'machine' milk and the 'machine' bagel each have two rates and two quantities associated with them.

► One might ask at this point how one should define a formal representation of a *Scheme* machine. This is one way: The formal representation/denotation of a *Scheme* machine is an ordered $(2n+1)$-tuple, whose first entry is the name of the machine, the next n entries are the names of the conversion factors (or rates), and the last n entries are their respective associated quantities. For example, a problem in which two pumps, P_1 and P_2, cooperate to fill a tank could be represented by (P_1, R_1, Q_1) and (P_2, R_2, Q_2).

But where would I come up with such a bizarre abstract 'machine' concept? Well, it happened to me quite naturally as I solved hundreds of 'difficult' word problems over the years. They seemed to all morph into each other. For example: What's really the difference between two pumps (real machines) actively filling a tank at two different rates **with** two drain pipes passively emptying a tank at two different rates **with** two printers printing a print job together at two different rates **with** two painters painting a house together at two different rates **with** forming a coffee blend by adding together two different coffees costing two different amounts per pound **with** alloying two different metals together that cost different amounts per pound **with** combining two piles of coins, one nickels, one dimes, into a single collection of coins, each pile adding quantities in different amounts and contributing dollar values at different rates, and so on?

The point being, *not* that there is some metaphysical 'machineness' intrinsic to all these examples of pairs of cooperating things, but that in every case listed above that is *not* a pair of cooperating real machines, the structure of the layout of the solution is the same as in the cases where they *are* two cooperating real machines. In other words, there's an analogy or isomorphism between them. (Similar to the point of category theory, is it?)

So, I state a rule that is already well known in mathematics, though I'm not aware that it has a name: There are times in the analysis of a problem that it's convenient to treat different things as the 'same' and to treat similar things as different. I can readily think of examples from combinatorics. I have an even simpler example: The claim that the operation of 3 apples + 2 oranges is meaningless is to emphasize the difference between apples and oranges, yet, to claim that the operation 3 apples + 2 oranges is meaningful and is equal to 5 pieces of fruit is to emphasize their commonality.

► Now, on to *Scheme* diagramming protocols: There are plenty of examples of how I diagram word problems in *Scheme* in the worked example problems presented in the beginning of this paper. The protocol is basically this: Entities are represented by boxes, whose names are in the boxes. Rates of change (conversion factors) are listed above the boxes and quantities are listed below the boxes. Rates of change include fractional amounts and ratios.[29] When a fractional amount is in the form of a part-to-whole, it has no units.

I found a similar diagramming protocol to that of *Scheme* at the website

`http://mgccc.edu/learning_lab/math/alg/howtomix.pdf`

One last point on *Scheme* diagrams: Although it seems to be the popular thing to do these days in both algebra word problems and stoichiometry to place variables and expressions in tabular form, I prefer not to. Obviously, I prefer the boxes-and-arrows approach. But there is one notable exception: That being the so-called 'age problem' and their generalization (the 'temporal' problem). In a typical age problem, two constituitive relations are given on the ages of two people, one on their current ages and another on their ages a given number of years in the past or in the future. The end result is immediately a system of two equations in two unknowns, which is ready to be solved.[30]

The following are problems that *Scheme* recommends using the tabular form of diagramming. This first one is found at

`https://www.algebra.com/algebra/homework/word/age/Age-problems-and-their-solutions.lesson`

▶ **Problem 1:** Kevin is 4 years older than Margaret.
Next year Kevin will be 2 times as old as Margaret.
How old is Kevin?

Solution: Use a tabular form of a diagram:

Person / Age	Kevin	Margaret	Constituitive Relations
Now	K	M	$K = M + 4$
Year from now	$K + 1$	$M + 1$	$(K + 1) = 2(M + 1)$

Figure C2. Using a tabular form for the diagram. The double lines are there to separate off the constituitive relations from the Person/Age information. Compared to a nontabular form of solution to this problem, the tabular form is definitely CNO (Clean, Neat, Organized), which aids in problem solving.

Regarding the diagram above, the two constituitive equations shown there *is* the system of equations to be solved for K and M, yielding, $K = 7$ and $M = 3$.

[29] The extension of *Scheme* to deal with stoichiometry adds a twist to this procedure: The *MoleStats line* is placed immediately below the boxes.

[30] I should also admit that I find the tabular form of the representation of variables and expressions in probability to be of great value to me.

Now, I'd like to say a word about how some people approach problems of this type. They will adopt what I call the *accelerated substitution* mode, whereby, they'll introduce the variable, K for Kevin's age, and then the variable $K-4$ for Margaret's age. This isn't technically wrong, of course, but I think it's confusing for beginners to learn. We shouldn't confuse for the beginner the distinction between the basic variables to a problem and the constituitive relations on those variables. Furthermore, accelerated substitution doesn't generalize well. For example, how does it work when the problem has three or more basic variables to solve for? However, I'm not totally against accelerated substitution. I just think that the student should learn the general concepts first, and the shortcuts later.

The next example problem I'd like to show is found at

`https://www.basic-mathematics.com/hard-word-problems-in-algebra.html`

Problem 99. Jacob's hourly wage is 4 times as much as Noah. When Jacob got a raise of 2 dollars Noah accepted a new position that pays him 2 dollars less per hour. Jacob now earns 5 times as much money as Noah. How much money do they make per hour after Jacob got the raise?

Solution

This kind of problem presents itself as a generalization of of the age problem. I'll refer to the class of problems that include this kind of problem and the age problems as *temporal problems*, where either time itself (ages, of course, have the units of time) or the main variables are functions of time.[31]

Let's begin with a tabular diagram:

	Jacob	Noah	Constituitive Relations
Beginning Wages	J	N	$J = 4N$
Ending Wages	$J + 2$	$N - 2$	$J + 2 = 5(N - 2)$

Figure C3. As a general rule, I avoid placing algebraic information into tables, but in this kind of problem (a 'temporal' problem), a table works very well.

As in the previous problem, we find the solution by simultaneously solving the constituitive relations presented in the diagram, obtaining (in units of dollars per hour) $J = 48$ and $N = 12$. Therefore, after the wages change:

$$J + 2 = 50\,, \quad N - 2 = 10\,. \tag{86}$$

[31]By 'functions of time' I do not mean to imply a continuous function of time, though it might be. For example, it could be that on a time line, all one knows is that something is true at one time and something else is true 'at a later time'.

Perhaps it is in contemplating the temporal problem type that the use of what I called the *naive approach* (see page 41) to solving algebra word problems became dominant, for it seems to work well for these kinds of problems.

As a final comment on temporal problems, I'd like to point out that in common parlance, we could categorize them as 'before-and-after' problems (which could cause some confusion). To obviate this, in *Scheme* this designation is used only for such problems as can generate a nontrivial equation of the form $Q_f = Q_i$, where Q is some invariant quantity of the process.

References

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